

Tunneling currents in ferromagnetic systems with multiple broken symmetries

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A system exhibiting multiple simultaneously broken symmetries offers the opportunity to influence physical phenomena such as tunneling currents by means of external control parameters. In this paper, we consider the broken $SU(2)$ (internal spin) symmetry of ferromagnetic systems coexisting with *i*) the broken $U(1)$ symmetry of superconductors and *ii*) the broken spatial inversion symmetry induced by a Rashba term in a spin-orbit coupling Hamiltonian. In order to study the effect of these broken symmetries, we consider tunneling currents that arise in two different systems; tunneling junctions consisting of non-unitary spin-triplet ferromagnetic superconductors and junctions consisting of ferromagnets with spin-orbit coupling. In the former case, we consider different pairing symmetries in a model where ferromagnetism and superconductivity coexist uniformly. An interplay between the relative magnetization orientation on each side of the junction and the superconducting phase difference is found, similarly to that found in earlier studies on spin-singlet superconductivity coexisting with spiral magnetism. This interplay gives rise to persistent spin- and charge-currents in the absence of an electrostatic voltage that can be controlled by adjusting the relative magnetization orientation on each side of the junction. In the second system, we study transport of spin in a system consisting of two ferromagnets with spin-orbit coupling separated by an insulating tunneling junction. A persistent spin-current across the junction is found, which can be controlled in a well-defined manner by external magnetic and electric fields. The behavior of this spin-current for important geometries and limits is studied.

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I. INTRODUCTION

Due to the increasing interest in the field of spintronics in recent years¹, the idea of utilizing the spin degree of freedom in electronic devices has triggered an extensive response in many scientific communities. The spin-Hall effect is arguably the research area which has received most focus in this context, with substantial effort being put into theoretical considerations² as well as experimental observations³. In spintronics, a main goal is to make use of the spin degree of freedom rather than electrical charge, investigations of mechanisms that offer ways of controlling spin-currents are of great interest. The study of systems with multiple broken symmetries is highly relevant in this context, since such systems promise rich physics with the opportunity to learn if the tunneling currents can be influenced by means of external control parameters such as electric and/or magnetic fields. Here, we will focus on two specific systems: ferromagnetism coexisting with superconductivity, which we shall refer to as ferromagnetic superconductors (FMSC), and systems where ferromagnetism and spin-orbit coupling are present (FMSO). In terms of broken symmetries, we will then study the broken $SU(2)$ (internal spin) symmetry of ferromagnetic systems coexisting with the broken $U(1)$ symmetry of superconductors and also consider ferromagnets with broken inversion (spatial) symmetry induced by a Rashba term in a spin-orbit coupling Hamiltonian.

The coexistence of ferromagnetism (FM) and superconductivity (SC) has a short history in experimental physics^{4,5,6}, although a theoretical proposition of this phenomenon was offered as early as 1957 by Ginzburg⁷. Spin-singlet superconductivity originating with BCS theory seems to be ruled out as a plausible pairing mechanism for a ferromagnetic

superconductor⁸, at least with regard to uniform coexistence of the FM and SC order parameters ζ and Δ , respectively. It could be achieved for a superconductor taking up a so-called Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state⁹. However, it seems likely that the coexistence of FM and SC call for^{10,11} p -wave spin-triplet Cooper pairs which have a non-zero magnetic moment. This type of pairing has been observed in superfluid ³He, and is perfectly compatible with FM order. Spin-triplet superconductivity has moreover been experimentally verified^{12,13} in Sr_2RuO_4 , and the study of such a pairing in a FMSC could unveil interesting effects with respect to quantum transport. The concept of simultaneously broken $U(1)$ and $SU(2)$ symmetries are of great interest from a fundamental physics point of view, and could be suggestive to a range of novel applications. This topic has been the subject of theoretical research in *e.g.* Refs. 14,15,16.

In this paper, we follow up Ref. 17 with a more comprehensive study of the tunneling currents between two p -wave FMSC separated by an insulating junction; $\text{RuSr}_2\text{GdCu}_2\text{O}_8$, UGe_2 , and URhGe have been proposed as candidates for such unconventional superconductors^{4,5,6}. In our model, we assume uniform coexistence of the FM and SC order parameters and that superconductivity arises from the same electrons that are responsible for the magnetism. As argued in Ref. 5, this can be understood most naturally as a spin-triplet rather than spin-singlet pairing phenomenon. Furthermore, it seems that SC in the metallic compounds mentioned above always coexists with the FM order and is enhanced by it¹⁸; the experiments conducted on the compounds UGe_2 and URhGe do not give any evidence for the existence of a standard normal-to-superconducting phase transition in a zero external magnetic field, but instead indicate a phase corresponding to a mixed state of FM and SC. We provide detailed calculations

for single-particle and Josephson (two-particle) tunneling between two non-unitary equal-spin pairing (ESP) FMSC. We examine both the charge- and spin-sector in detail within linear response theory using the Kubo formula. We find that the supercurrent of spin and charge may be controlled by adjusting the misorientation of the exchange fields on both sides of the junction. Such an effect was first discovered by Kulic and Kulic¹⁹, who derived an expression for the Josephson current over a junction separating two BCS superconductors with spiral magnetic order. It was found that the supercurrent could be controlled by adjusting the relative orientation of the exchange field on both sides of the junction, a finding that quite remarkably suggested a way of tuning a supercurrent in a well-defined manner from *e.g.* a 0- to π -junction. Later investigations made by Eremin, Nogueira, and Tarento²⁰ considered a similar system as Kulic and Kulic¹⁹, namely two Fulde-Ferrel-Larkin-Ovchinnikov (FFLO) superconductors⁹ coexisting with helimagnetic order. Recently, the same opportunity was found to exist in a FMSC/I/FMSC junction as shown by Grønsløth *et al.*¹⁷.

In the case of a system where both ferromagnetism and spin-orbit coupling are present, it is clear that these are physical properties of a system that crucially influence the behavior of spins present in that system. For instance, the presence of spin-orbit coupling is highly important when considering ferromagnetic semiconductors^{21,22}. Such materials have been proposed as devices for obtaining controllable spin injection and manipulating single electron spins by means of external electrical fields, making them a central topic of semiconductor spintronics²³. In ferromagnetic metals, spin-orbit coupling is ordinarily significantly smaller than for semiconductors due to the bandstructure. However, the presence of a spin-orbit coupling in ferromagnets could lead to new effects in terms of quantum transport.

Studies of tunneling between ferromagnets have uncovered interesting physical effects^{24,25,26}. Nogueira *et al.* predicted²⁴ that a dissipationless spin-current should be established across the junction of two Heisenberg ferromagnets, and that the spin-current was maximal in the special case of tunneling between planar ferromagnets. Also, there has been investigations of what kind of impact spin-orbit coupling constitutes on tunneling currents in various contexts, *e.g.* for noncentrosymmetric superconductors²⁷, and two-dimensional electron gases coupled to ferromagnets²⁸. Broken time reversal and inversion-symmetry are interesting properties of a system with regard to quantum transport of spin and charge, and the exploitation of such asymmetries has given rise to several devices in recent years. For instance, the broken $SU(2)$ symmetry exhibited by ferromagnets has a broad range of possible applications. This has led to spin current induced magnetization switching²⁹, and suggestions have been made for more exotic devices such as spin-torque transistors³⁰ and spin-batteries³¹. It has also led to investigations into such phenomena as spin-Hall effect in paramagnetic metals³², spin-pumping from ferromagnets into metals, enhanced damping of spins when spins are pumped from one ferromagnet to another through a metallic sample³³, and the mentioned spin Josephson effects in ferromagnet/ferromagnet tunneling

junctions²⁴.

Here, we study the spin-current that arises over a tunneling junction separating two ferromagnetic metals with substantial spin-orbit coupling. It is found that the total current consists of three terms; one due to a twist in magnetization across the junction (in agreement with the result of Ref. 24), one term originating from the spin-orbit interactions in the system, and finally an interesting mixed term that stems from an interplay between the ferromagnetism and spin-orbit coupling. After deriving the expression for the spin-current between Heisenberg ferromagnets with substantial spin-orbit coupling, we consider important tunneling geometries and physical limits of our generally valid results. Finally, we make suggestions concerning the detection of the predicted spin-current. Our results indicate how spin transport between systems exhibiting both magnetism and spin-orbit coupling can be controlled by external fields, and should therefore be of considerable interest in terms of spintronics.

This paper is organized as follows. In Sec. II, we consider transport between spin-triplet ferromagnetic superconductors, while a study of transport between ferromagnets with spin-orbit coupling is given in Sec. III. A discussion of our results is provided in Sec. IV, with emphasis on how the novel effects predicted in this paper could be tested in an experimental setup. Finally, we give concluding remarks in Sec. V.

II. FERROMAGNETIC SUPERCONDUCTORS

A. Coexistence of ferromagnetism and superconductivity

An important issue to address concerning FMSC is whether the SC and FM order parameters coexist uniformly or if they are phase-separated. One possibility³⁴ is that a spontaneously formed vortex lattice due to the internal magnetization \mathbf{m} is realized in a spin-triplet FMSC, while there also have been studies of Meissner (uniform) SC phases in spin-triplet FMSC¹⁸. As argued in Ref. 35, a key variable with respect to whether a vortex lattice appears or not is the strength of the internal magnetization \mathbf{m} . Ref. 36 suggested that vortices arise if $4\pi\mathbf{m} > \mathbf{H}_{c1}$, where \mathbf{H}_{c1} is the lower critical field. When considering a weak FM state coexisting with SC, a scenario which seems to be the case for URhGe, the domain structure in the absence of an external field is thus vortex-free. Current experimental data concerning URhGe are not strong enough to unambiguously settle this question, while evidence for uniform coexistence of FM and SC has been indicated³⁷ in UGe₂. Furthermore, a bulk Meissner state in the FMSC RuSr₂GdCu₂O₈ has been reported in Ref. 38, hence suggesting the existence of uniform FM and SC as a bulk effect. In our study, we shall consequently take the order parameters as coexisting homogeneously and use their bulk values, as justified by the argumentation above. However, we emphasize that one in general should take into account the possible suppression of the SC order parameter in the vicinity of the tunneling interface due to the formation of midgap surface states³⁹ which occur for certain orientations of the SC gap. The pair-breaking effect of these states in unconventional superconductors

tors has been studied in e.g.^{40,41,42}, and we discuss this in more detail in Sec. IV. A sizeable formation of such states would suppress the Josephson current, although it is nonvanishing in the general case. Also, we use bulk uniform magnetic order parameters, as in²⁴. The latter is justified on the grounds that a ferromagnet with a planar order parameter is mathematically isomorphic to an s -wave superconductor, where the use of bulk values for the order parameter right up to the interface is a good approximation due to the lack of midgap surface states.

It is generally believed that the same electrons that are responsible for itinerant FM also participate in the formation of Cooper pairs below the SC critical temperature⁶. As a consequence, uniform coexistence of spin-singlet SC and FM can be discarded since s -wave Cooper pairs carry a total spin of zero, although spatially modulated order parameters could allow for magnetic s -wave superconductors^{19,20}. However, spin-triplet Cooper pairs are in principle perfectly compatible with FM order since they can carry a net magnetic moment. To see this, consider the $\mathbf{d}_\mathbf{k}$ -vector formalism⁴³ which is convenient when dealing with spin-triplet superconductors, regardless of whether they are magnetic or not. For a complete and rigorous treatment of the $\mathbf{d}_\mathbf{k}$ -vector order parameter, see e.g. Ref. 44. The spin dependence of triplet pairing can be represented by a 2×2 matrix

$$\hat{\Delta}_\mathbf{k} = \begin{pmatrix} \Delta_{\mathbf{k}\uparrow\uparrow} & \Delta_{\mathbf{k}\uparrow\downarrow} \\ \Delta_{\mathbf{k}\downarrow\uparrow} & \Delta_{\mathbf{k}\downarrow\downarrow} \end{pmatrix} \quad (1)$$

$$= \begin{pmatrix} -d_x(\mathbf{k}) + id_y(\mathbf{k}) & d_z(\mathbf{k}) \\ d_z(\mathbf{k}) & d_x(\mathbf{k}) + id_y(\mathbf{k}) \end{pmatrix} = i\mathbf{d}_\mathbf{k} \cdot \hat{\sigma} \hat{\sigma}_y,$$

where $\Delta_{\mathbf{k}\alpha\beta}$ represent the SC gap parameters for different triplet pairings, $\hat{\sigma} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$ where $\hat{\sigma}_i$ are the Pauli matrices, and $\mathbf{d}_\mathbf{k} = (d_x(\mathbf{k}), d_y(\mathbf{k}), d_z(\mathbf{k}))$ is given by

$$\mathbf{d}_\mathbf{k} = \left(\frac{\Delta_{\mathbf{k}\downarrow\downarrow} - \Delta_{\mathbf{k}\uparrow\uparrow}}{2}, -i \frac{(\Delta_{\mathbf{k}\downarrow\downarrow} + \Delta_{\mathbf{k}\uparrow\uparrow})}{2}, \Delta_{\mathbf{k}\uparrow\downarrow} \right). \quad (2)$$

Note that $\mathbf{d}_\mathbf{k}$ transforms like a vector under spin rotations and that $\Delta_{\mathbf{k}\uparrow\downarrow} = \Delta_{\mathbf{k}\downarrow\uparrow}$ for triplet pairing since it is of no significance which electron in the Cooper pair that has spin up or down. This is because spin-part of the two-particle wavefunction is symmetric under exchange of particles, as opposed to spin-singlet SC, where the gap changes sign when the spin indices are exchanged. Spin-triplet SC states are classified as unitary if $i\mathbf{d}_\mathbf{k} \times \mathbf{d}_\mathbf{k}^* = 0$ and non-unitary if the equality sign does not hold. Since the average spin of a $\mathbf{d}_\mathbf{k}$ -state is given by⁴⁴

$$\langle \mathbf{S}_\mathbf{k} \rangle = i\mathbf{d}_\mathbf{k} \times \mathbf{d}_\mathbf{k}^*, \quad (3)$$

it is clear that we must have a non-unitary $\mathbf{d}_\mathbf{k}$ in a model where FM and SC coexist uniformly. Indeed, there is strong reason to believe that the correct pairing symmetries in the discovered FMSC constitute non-unitary states^{11,45,46}. As a consequence, one can rule out for instance a state where only $\Delta_{\mathbf{k}\uparrow\downarrow} \neq 0$ since it would imply $\langle \mathbf{S}_\mathbf{k} \rangle = 0$ according to Eq. (3). In the most general case where all SC gaps are included, $\Delta_{\uparrow\downarrow}$ would be suppressed in the presence of a Zeeman-splitting between the \uparrow, \downarrow conduction bands⁶; see Fig. 1.

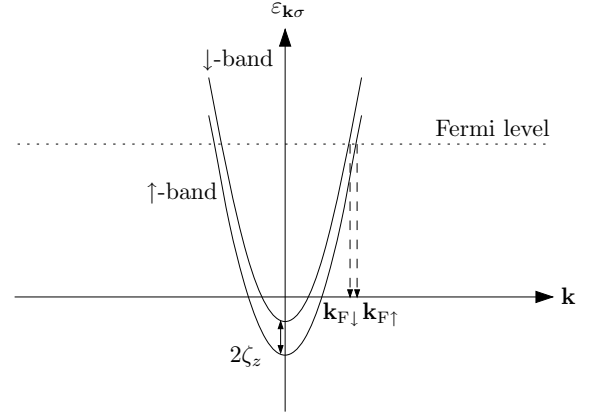


FIG. 1: Band-splitting for \uparrow, \downarrow electrons in the presence of a magnetization in \hat{z} -direction. Inter-band pairing gives rise to a net Cooper pair momentum in the presence of a band-splitting, thus suppressing the $\Delta_{\mathbf{k}\uparrow\downarrow}$ order parameter.

However, such a splitting between energy-bands need not be present and one could in theory then consider a \mathbf{d} -vector where

$$|\Delta_{\mathbf{k}\uparrow\uparrow}| = |\Delta_{\mathbf{k}\downarrow\downarrow}| \neq 0, \Delta_{\mathbf{k}\uparrow\downarrow} \neq 0 \quad (4)$$

such that $\langle \mathbf{S}_\mathbf{k} \rangle$ lies in the local xy -plane. This scenario would be equivalent to an A2-phase as is seen when performing a spin rotation on the gap parameters into a quantization axis lying in the xy -plane. Denoting up- and down-spins with respect to the new quantization axis by $+$ and $-$, respectively, the transformation yields

$$\begin{pmatrix} \Delta_{\mathbf{k}\uparrow\uparrow} \\ \Delta_{\mathbf{k}\uparrow\downarrow} \\ \Delta_{\mathbf{k}\downarrow\downarrow} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 2e^{i\phi} & e^{2i\phi} \\ -e^{-i\phi} & 0 & e^{i\phi} \\ e^{-2i\phi} & -2e^{-i\phi} & 1 \end{pmatrix} \begin{pmatrix} \tilde{\Delta}_{\mathbf{k}++} \\ \tilde{\Delta}_{\mathbf{k}+-} \\ \tilde{\Delta}_{\mathbf{k}--} \end{pmatrix}, \quad (5)$$

where ϕ is the azimuthal angle as shown in Fig. 2.

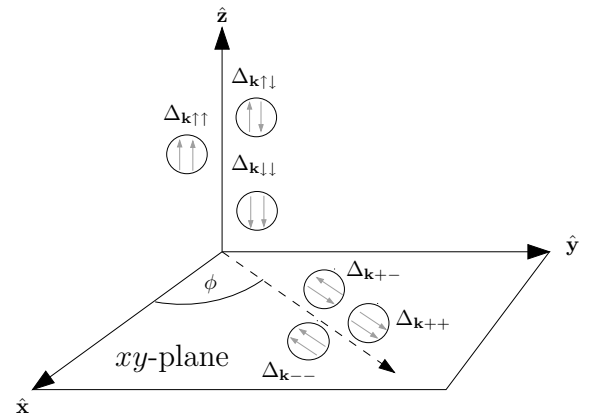


FIG. 2: Change of spin-basis for the superconducting gaps. The new quantization axis is represented by the dotted arrow.

When introducing the conditions in Eq. (4), it is readily seen that $\tilde{\Delta}_{\mathbf{k}+-} = 0$ while $|\tilde{\Delta}_{\mathbf{k}++}| \neq |\tilde{\Delta}_{\mathbf{k}--}| \neq 0$, thus

corresponding to an A2-phase. Consequently, the entire span of physically possible pairing symmetries in a FMSC can be reduced to the equivalence of an A1- or A2-phase in ^3He by a change of spin-basis. The definitions of A-, A1-, and A2-phases in ^3He are as follow: an A-phase corresponds to a pairing symmetry such that $|\Delta_{\mathbf{k}\uparrow\uparrow}| = |\Delta_{\mathbf{k}\downarrow\downarrow}| \neq 0$, an A1-phase has only one gap $\Delta_{\mathbf{k}\sigma\sigma} \neq 0$ while $\Delta_{\mathbf{k},-\sigma,-\sigma} = 0$, and an A2-phase satisfies $|\Delta_{\mathbf{k}\uparrow\uparrow}| \neq |\Delta_{\mathbf{k}\downarrow\downarrow}| \neq 0$. In this case, $\Delta_{\mathbf{k}\alpha\beta}$ represents the superfluid gap for the fermionic ^3He -atoms, and $\Delta_{\mathbf{k}\uparrow\downarrow} = 0$ for all A_i-phases.

The resulting spin of the Cooper pair is then in general given by

$$\langle \mathbf{S}_{\mathbf{k}} \rangle = (1/2)[|\Delta_{\mathbf{k}\uparrow\uparrow}|^2 - |\Delta_{\mathbf{k}\downarrow\downarrow}|^2]\hat{\mathbf{z}}. \quad (6)$$

In the following, we shall accordingly consider tunneling between non-unitary ESP FMSC in an A1- or A2-phase. Moreover, we consider thin film FMSC, ensuring that no accumulation of charge at the surface will take place due to an orbital effect. Our system can be thought to have arisen by first cooling down a sample below the Curie temperature T_M such that FM order is introduced. At further cooling below the critical temperature T_c , the same electrons that give rise to FM condense into Cooper pairs with a net magnetic moment parallel to the original direction of magnetization. Our model is shown in Fig. 3.

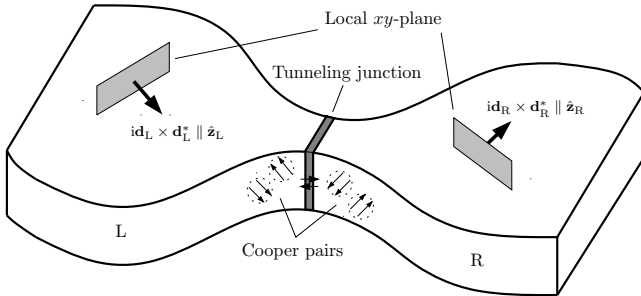


FIG. 3: Tunneling between two non-unitary ESP FMSC. The quantization axis has been taken along the direction of magnetization on each side of the junction.

B. The Hamiltonian

The system consists of two FMSC separated by an insulating layer such that the total Hamiltonian can be written as⁴⁷ $H = H_L + H_R + H_T$, where L and R represents the individual FMSC on each side of the tunneling junction, and H_T describes tunneling of particles through the insulating layer separating the two pieces of bulk material. Using mean-field theory, one finds that the individual FMSC are described by a

Hamiltonian similar to the one used in Ref. 48,

$$H_{\text{FMSC}} = H_0 + \sum_{\mathbf{k}} \psi_{\mathbf{k}}^\dagger \hat{A}_{\mathbf{k}} \psi_{\mathbf{k}},$$

$$H_0 = JN\eta(0)\mathbf{m}^2 + \frac{1}{2} \sum_{\mathbf{k}\sigma} \varepsilon_{\mathbf{k}\sigma} + \sum_{\mathbf{k}\alpha\beta} \Delta_{\mathbf{k}\alpha\beta}^\dagger b_{\mathbf{k}\alpha\beta}. \quad (7)$$

Here, \mathbf{k} is the electron momentum and we have introduced

$$\varepsilon_{\mathbf{k}\sigma} = \varepsilon_{\mathbf{k}} - \sigma\zeta_z, \quad \sigma = \uparrow, \downarrow = \pm 1. \quad (8)$$

Furthermore, J is a spin coupling constant, $\eta(\mathbf{k})$ is a geometrical structure factor which for $\mathbf{k} = 0$ reduces to the number of nearest lattice neighbors $\eta(0)$, $\mathbf{m} = \{m_x, m_y, m_z\}$ is the magnetization vector, while $\Delta_{\mathbf{k}\alpha\beta}$ is the superconducting order parameter and $b_{\mathbf{k}\alpha\beta} = \langle c_{-\mathbf{k}\beta} c_{\mathbf{k}\alpha} \rangle$ denotes the two-particle operator expectation value. The ferromagnetic order parameters are given by

$$\zeta = 2J\eta(0)(m_x - im_y), \quad \zeta_z = 2J\eta(0)m_z. \quad (9)$$

The interesting physics of the FMSC/FMSC junction lies in the matrix $\hat{A}_{\mathbf{k}}$ to be given below. Above, we used a basis

$$\psi_{\mathbf{k}} = (c_{\mathbf{k}\uparrow} \ c_{\mathbf{k}\downarrow} \ c_{-\mathbf{k}\uparrow}^\dagger \ c_{-\mathbf{k}\downarrow}^\dagger)^T, \quad (10)$$

where $c_{\mathbf{k}\sigma}$ ($c_{\mathbf{k}\sigma}^\dagger$) are annihilation (creation) fermion operators. Note that we have not incorporated any spin-orbit coupling of the type $(\mathbf{E} \times \mathbf{p}) \cdot \hat{\sigma}$ in the Hamiltonian described in Eq. (7) such that spatial inversion symmetry is not broken, *i.e.* we consider centrosymmetric FMSC.

Consider now the matrix

$$\hat{A}_{\mathbf{k}} = -\frac{1}{2} \begin{pmatrix} -\varepsilon_{\mathbf{k}\uparrow} & \zeta & \Delta_{\mathbf{k}\uparrow\uparrow} & \Delta_{\mathbf{k}\uparrow\downarrow} \\ \zeta^\dagger & -\varepsilon_{\mathbf{k}\downarrow} & \Delta_{\mathbf{k}\downarrow\uparrow} & \Delta_{\mathbf{k}\downarrow\downarrow} \\ \Delta_{\mathbf{k}\uparrow\uparrow}^\dagger & \Delta_{\mathbf{k}\uparrow\downarrow}^\dagger & \varepsilon_{\mathbf{k}\uparrow} & -\zeta^\dagger \\ \Delta_{\mathbf{k}\uparrow\downarrow}^\dagger & \Delta_{\mathbf{k}\downarrow\downarrow}^\dagger & -\zeta & \varepsilon_{\mathbf{k}\downarrow} \end{pmatrix}, \quad (11)$$

which is valid for a FMSC with arbitrary magnetization. As explained in the previous sections, we will study in detail tunneling between non-unitary ESP FMSC, *i.e.* $\Delta_{\mathbf{k}\uparrow\downarrow} = \Delta_{\mathbf{k}\downarrow\uparrow} = 0$, $\zeta = 0$ in Eq. (11). We take the quantization axis on each side of the junction to coincide with the magnetization direction. One then needs to include the Wigner d -function⁴⁹ denoted by $\hat{\mathcal{D}}_{\sigma'\sigma}^{(j)}(\vartheta)$ with $j = 1/2$ to account for the fact that a \uparrow spin on one side of the junction is not the same as a \uparrow spin on the other side of the junction, since the magnetization vectors point can point in different directions. The angle ϑ is consequently defined by

$$\mathbf{m}_R \cdot \mathbf{m}_L = m_R m_L \cos(\vartheta), \quad m_i = |\mathbf{m}_i|. \quad (12)$$

Specifically, we have that

$$\hat{\mathcal{D}}^{(1/2)}(\vartheta) = \begin{pmatrix} \cos(\vartheta/2) & -\sin(\vartheta/2) \\ \sin(\vartheta/2) & \cos(\vartheta/2) \end{pmatrix} \quad (13)$$

such that a spin-rotated fermion operator is given by

$$\tilde{d}_{\mathbf{p}\sigma} = \sum_{\sigma'} \hat{\mathcal{D}}_{\sigma'\sigma}^{(1/2)}(\vartheta) d_{\mathbf{p}\sigma'}. \quad (14)$$

The tunneling Hamiltonian then reads

$$H_T = \sum_{\mathbf{k}\mathbf{p}\sigma\sigma'} \hat{\mathcal{D}}_{\sigma'\sigma}^{(1/2)}(\vartheta) \left(T_{\mathbf{k}\mathbf{p}} c_{\mathbf{k}\sigma}^\dagger d_{\mathbf{p}\sigma'} + T_{\mathbf{k}\mathbf{p}}^* d_{\mathbf{p}\sigma'}^\dagger c_{\mathbf{k}\sigma} \right), \quad (15)$$

where we neglect the possibility of spin-flips in the tunneling process. Note that we distinguish between fermion operators on the right and left side of the junction corresponding to $c_{\mathbf{k}\sigma}$ and $d_{\mathbf{p}\sigma}$, respectively. Demanding that H_T is invariant under time reversal \mathcal{K} , one finds that the condition $\mathcal{K}^{-1} H_T \mathcal{K} = H_T$ with

$$\mathcal{K}^{-1} H_T \mathcal{K} = \sum_{\mathbf{k}\mathbf{p}\sigma\sigma'} \sigma\sigma' \hat{\mathcal{D}}_{\sigma'\sigma}^{(1/2)}(\vartheta) \times \left(T_{\mathbf{k}\mathbf{p}}^* c_{-\mathbf{k},-\sigma}^\dagger d_{-\mathbf{p},-\sigma'} + T_{\mathbf{k}\mathbf{p}} d_{-\mathbf{p},-\sigma'}^\dagger c_{-\mathbf{k},-\sigma} \right) \quad (16)$$

dictates that $T_{\mathbf{k}\mathbf{p}} = T_{-\mathbf{k},-\mathbf{p}}^*$. Furthermore, we write the superconducting order parameters as $\Delta_{\mathbf{k}\sigma\sigma} = |\Delta_{\mathbf{k}\sigma\sigma}| e^{i(\theta_{\mathbf{k}} + \theta_{\sigma\sigma}^R)}$, where R (L) denotes the bulk superconducting phase on the right (left) side of the junction while $\theta_{\mathbf{k}}$ is a general (complex) internal phase factor originating from the specific form of the gap in \mathbf{k} -space that ensures odd symmetry under inversion of momentum, *i.e.* $\theta_{\mathbf{k}} = \theta_{-\mathbf{k}} + \pi$.

For our system, Eq. (7) takes the form

$$H_{\text{FMSC}} = H_0 + H_A, \quad H_A = \sum_{\mathbf{k}\sigma} \phi_{\mathbf{k}\sigma}^\dagger \hat{A}_{\mathbf{k}\sigma} \phi_{\mathbf{k}\sigma}, \quad (17)$$

where we have block-diagonalized $\hat{A}_{\mathbf{k}}$ and chosen a convenient basis $\phi_{\mathbf{k}\sigma}^\dagger = (c_{\mathbf{k}\sigma}^\dagger, c_{-\mathbf{k}\sigma}^\dagger)$, with the definition

$$\hat{A}_{\mathbf{k}\sigma} = -\frac{1}{2} \begin{pmatrix} -\varepsilon_{\mathbf{k}\sigma} & \Delta_{\mathbf{k}\sigma\sigma} \\ \Delta_{\mathbf{k}\sigma\sigma}^\dagger & \varepsilon_{\mathbf{k}\sigma} \end{pmatrix}. \quad (18)$$

This Hamiltonian is diagonalized by a 2×2 spin generalized unitary matrix $\hat{U}_{\mathbf{k}\sigma}$, so that the superconducting sector is expressed in the diagonal basis

$$\tilde{\phi}_{\mathbf{k}\sigma}^\dagger = \phi_{\mathbf{k}\sigma}^\dagger \hat{U}_{\mathbf{k}\sigma} \equiv (\gamma_{\mathbf{k}\sigma}^\dagger, \gamma_{-\mathbf{k}\sigma}^\dagger). \quad (19)$$

Thus, $H_A = \sum_{\mathbf{k}\sigma} \tilde{\phi}_{\mathbf{k}\sigma}^\dagger \hat{A}_{\mathbf{k}\sigma} \tilde{\phi}_{\mathbf{k}\sigma}$, in which

$$\hat{A}_{\mathbf{k}\sigma} = \hat{U}_{\mathbf{k}\sigma} \hat{A}_{\mathbf{k}\sigma} \hat{U}_{\mathbf{k}\sigma}^{-1} = \text{diag}(\tilde{E}_{\mathbf{k}\sigma}, -\tilde{E}_{\mathbf{k}\sigma})/2, \quad \tilde{E}_{\mathbf{k}\sigma} = \sqrt{\varepsilon_{\mathbf{k}\sigma}^2 + |\Delta_{\mathbf{k}\sigma\sigma}|^2}. \quad (20)$$

The explicit expression for $\hat{U}_{\mathbf{k}\sigma}$ is

$$\hat{U}_{\mathbf{k}\sigma} = N_{\mathbf{k}\sigma} \begin{pmatrix} 1 & \frac{\Delta_{\mathbf{k}\sigma\sigma}}{\varepsilon_{\mathbf{k}\sigma} + E_{\mathbf{k}\sigma}} \\ -\frac{\Delta_{\mathbf{k}\sigma\sigma}^*}{\varepsilon_{\mathbf{k}\sigma} + E_{\mathbf{k}\sigma}} & 1 \end{pmatrix}, \quad N_{\mathbf{k}\sigma} = \frac{\varepsilon_{\mathbf{k}\sigma} + \tilde{E}_{\mathbf{k}\sigma}}{\sqrt{(\varepsilon_{\mathbf{k}\sigma} + \tilde{E}_{\mathbf{k}\sigma})^2 + |\Delta_{\mathbf{k}\sigma\sigma}|^2}}. \quad (21)$$

We now proceed to investigate the tunneling currents that can arise across a junction of two such FMSC.

C. Tunneling formalism

Although the treatment in this section is fairly standard, it comes with certain extension to the standard cases due to the coexistence of two simultaneously broken symmetries. Thus, for completeness, we present it here.

In order to find the spin- and charge-current over the junction, we define the generalized number operator⁶⁷ by $N_{\alpha\beta} = \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\beta}$. Consider now the transport operator

$$\begin{aligned} \dot{N}_{\alpha\beta} &= i[H_T, N_{\alpha\beta}] \\ &= -i \sum_{\mathbf{k}\mathbf{p}\sigma} [\hat{\mathcal{D}}_{\sigma\beta}^{(1/2)}(\vartheta) T_{\mathbf{k}\mathbf{p}} c_{\mathbf{k}\alpha}^\dagger d_{\mathbf{p}\sigma} - \hat{\mathcal{D}}_{\sigma\alpha}^{(1/2)}(\vartheta) T_{\mathbf{k}\mathbf{p}}^* d_{\mathbf{p}\sigma}^\dagger c_{\mathbf{k}\beta}]. \end{aligned} \quad (22)$$

We now write $H = H' + H_T$ where $H' = H_L + H_R$ and $H_i = K_i + \mu_i N_i$, $i = L, R$, where μ_i is the chemical potential on side i and N_i is the number operator. In the interaction picture, the time-dependence of $\dot{N}_{\alpha\beta}$ is then governed by

$$\dot{N}_{\alpha\beta}(t) = e^{iH't} \dot{N}_{\alpha\beta} e^{-iH't}, \quad (23)$$

while the time-dependence of the fermion operators reads

$$c_{\mathbf{k}\sigma}(t) = e^{iK_R t} c_{\mathbf{k}\sigma} e^{-iK_R t}. \quad (24)$$

Effectively, one can write

$$K_R = H_0 + \sum_{\mathbf{k}\sigma} E_{\mathbf{k}\sigma} \gamma_{\mathbf{k}\sigma}^\dagger \gamma_{\mathbf{k}\sigma}, \quad (25)$$

where the chemical potential is now included in the quasi-particle excitation energies $E_{\mathbf{k}\sigma}$ according to

$$E_{\mathbf{k}\sigma} = \sqrt{\xi_{\mathbf{k}\sigma}^2 + |\Delta_{\mathbf{k}\sigma\sigma}|^2} \quad (26)$$

with $\xi_{\mathbf{k}\sigma} = \varepsilon_{\mathbf{k}\sigma} - \mu_R$, and correspondingly for the left side. Consequently, we are able to write down

$$\begin{aligned} \dot{N}_{\alpha\beta}(t) &= -i \sum_{\mathbf{k}\mathbf{p}\sigma} \left(\hat{\mathcal{D}}_{\sigma\beta}^{(1/2)}(\vartheta) T_{\mathbf{k}\mathbf{p}} c_{\mathbf{k}\alpha}^\dagger(t) d_{\mathbf{p}\sigma}(t) e^{-iteV} \right. \\ &\quad \left. - \hat{\mathcal{D}}_{\sigma\alpha}^{(1/2)}(\vartheta) T_{\mathbf{k}\mathbf{p}}^* d_{\mathbf{p}\sigma}^\dagger(t) c_{\mathbf{k}\beta}(t) e^{iteV} \right), \end{aligned} \quad (27)$$

where $eV \equiv \mu_L - \mu_R$ is the externally applied potential. Within linear response theory, we can identify a general current

$$\mathbf{I}(t) = \sum_{\alpha\beta} \hat{\tau}_{\alpha\beta} \langle \dot{N}_{\alpha\beta}(t) \rangle, \quad \hat{\tau} = (-e\hat{1}, \hat{\sigma}), \quad (28)$$

such that the charge-current is $I^C(t) = I_0(t)$ while the spin-current reads $\mathbf{I}^S(t) = (I_1(t), I_2(t), I_3(t))$. In Eq. (28), $\hat{1}$ denotes the 2×2 identity matrix. Explicitly, we have

$$\begin{aligned} I^C(t) &= I_{\text{sp}}^C(t) + I_{\text{ip}}^C(t) = -e \sum_{\alpha} \langle \dot{N}_{\alpha\alpha}(t) \rangle \\ \mathbf{I}^S(t) &= \mathbf{I}_{\text{sp}}^S(t) + \mathbf{I}_{\text{ip}}^S(t) = \sum_{\alpha\beta} \hat{\sigma}_{\alpha\beta} \langle \dot{N}_{\alpha\beta}(t) \rangle, \end{aligned} \quad (29)$$

where the subscripts sp and tp denote the single-particle and two-particle contribution to the currents, respectively. As recently pointed out by the authors of Ref. 50, defining a spin-current is not as straight-forward as defining a charge-current. Specifically, the conventional definition of a spin-current given as spin multiplied with velocity suffers from severe flaws in systems where spin is not a conserved quantity. In this paper, we define the spin-current across the junction as $\mathbf{I}^S(t) = \langle d\mathbf{S}(t)/dt \rangle$ where $d\mathbf{S}/dt = \mathbf{I}[H_T, \mathbf{S}]$. It is then clear that the concept of a spin-current in this context refers to the rate at which the spin-vector \mathbf{S} on one side of the junction changes *as a result of tunneling across the junction*. The spatial components of \mathbf{I}^S are defined with respect to the corresponding quantization axis. In this way, we avoid non-physical interpretations of the spin-current in terms of real spin transport as we only calculate the contribution to $d\mathbf{S}/dt$ from the tunneling Hamiltonian *instead* of the entire Hamiltonian H . Should we have chosen the latter approach, one would in general run the risk of obtaining a non-zero spin-current due to *e.g.* local spin-flip processes which are obviously not relevant in terms of real spin transport across the junction. However, in our system such spin-flip processes are absent.

The tunneling currents are calculated in the linear response regime by using the Kubo formula,

$$\langle \dot{N}_{\alpha\beta}(t) \rangle = -i \int_{-\infty}^t dt' \langle [\dot{N}_{\alpha\beta}(t), H_T(t')] \rangle, \quad (30)$$

where the right hand side is the statistical expectation value in the unperturbed quantum state, *i.e.* when the two subsystems are not coupled. This expression includes both single-particle and two-particle contributions to the current. Details of the calculations are found in Sec. A 1.

We now consider the cases of an A2- and A1-phase at zero external potential, giving special attention to the charge-current and \hat{z} -component of the spin-current in the Josephson channel.

D. Two-particle currents

For an A2-phase in the case of zero externally applied voltage ($eV = 0$), Eqs. (29) and (A13) generates a quasiparticle interference term I_{qi} , in addition to a term I_J identified as the Josephson current. Thus, the total two-particle currents of charge and spin can be written as $I_{tp(z)}^{C(S)} = I_{qi(z)}^{C(S)} + I_{J(z)}^{C(S)}$ where

$$\begin{aligned} I_{qi(z)}^{C(S)} &= \sum_{\mathbf{k}\mathbf{p}} I^{C(S)}(\theta_{\sigma\sigma}^L - \theta_{\alpha\alpha}^R, \Delta\theta_{\mathbf{p}\mathbf{k}}), \\ I_{J(z)}^{C(S)} &= \sum_{\mathbf{k}\mathbf{p}} I^{C(S)}(\Delta\theta_{\mathbf{p}\mathbf{k}}, \theta_{\sigma\sigma}^L - \theta_{\alpha\alpha}^R) \end{aligned} \quad (31)$$

with the definitions

$$\begin{aligned} I^C(\phi_1, \phi_2) &= \frac{e}{2} \sum_{\sigma\alpha} [1 + \sigma\alpha \cos(\vartheta)] |T_{\mathbf{k}\mathbf{p}}|^2 \frac{|\Delta_{\mathbf{k}\alpha\alpha}| |\Delta_{\mathbf{p}\sigma\sigma}|}{E_{\mathbf{k}\alpha} E_{\mathbf{p}\sigma}} \\ &\quad \times \cos(\phi_1) \sin(\phi_2) F_{\mathbf{k}\mathbf{p}\alpha\sigma}, \\ I^S(\phi_1, \phi_2) &= -\frac{1}{2} \sum_{\sigma\alpha} \alpha [1 + \sigma\alpha \cos(\vartheta)] |T_{\mathbf{k}\mathbf{p}}|^2 \frac{|\Delta_{\mathbf{k}\alpha\alpha}| |\Delta_{\mathbf{p}\sigma\sigma}|}{E_{\mathbf{k}\alpha} E_{\mathbf{p}\sigma}} \\ &\quad \times \cos(\phi_1) \sin(\phi_2) F_{\mathbf{k}\mathbf{p}\alpha\sigma}, \end{aligned} \quad (32)$$

where we have introduced $\Delta\theta_{\mathbf{p}\mathbf{k}} \equiv \theta_{\mathbf{p}} - \theta_{\mathbf{k}}$ and

$$F_{\mathbf{k}\mathbf{p}\alpha\sigma} = \sum_{\pm} \frac{f(\pm E_{\mathbf{k}\alpha}) - f(E_{\mathbf{p}\sigma})}{E_{\mathbf{k}\alpha} \mp E_{\mathbf{p}\sigma}}. \quad (33)$$

Above, $f(x)$ is the Fermi distribution. Thus, we have found a two-particle current, for both spin and charge, that can be tuned in a well-defined manner by adjusting the relative orientation ϑ of the magnetization vectors⁶⁸. We will discuss the detection of such an effect later in this paper. Note that the \mathbf{k} -dependent symmetry factor $\theta_{\mathbf{k}}$ enters the above expressions, thus giving rise to an extra contribution to the two-particle current besides the ordinary Josephson effect. This is due to the fact that we included it in the SC gaps as a factor $e^{i\theta_{\mathbf{k}}}$ which in general is complex. However, this specific form may for certain models, depending on the Fermi surface in question, be reduced to a real function, *i.e.* $e^{i\theta_{\mathbf{k}}} \rightarrow \cos \theta_{\mathbf{k}}$, in which case the quasi-particle interference term becomes zero. Hence, in most of the remaining discussion we will focus on the Josephson part of the two-particle current.

The A1-phase with only one SC order parameter $\Delta_{\mathbf{k}\alpha\alpha}$, $\alpha \in \{\uparrow, \downarrow\}$ also corresponds to a non-unitary state $\mathbf{d}_{\mathbf{k}}$ according to Eq. (3), and is thus compatible with coexistence of FM and SC. In this case, we readily see that Eq. (32) reduces to

$$\begin{aligned} I_{tp}^C &= e \cos^2(\vartheta/2) X_{\alpha} \\ I_{tp,z}^S &= -\alpha \cos^2(\vartheta/2) X_{\alpha} \end{aligned} \quad \alpha \in \{\uparrow, \downarrow\} \quad (34)$$

where we have defined the quantity

$$\begin{aligned} X_{\alpha} &= \sum_{\mathbf{k}\mathbf{p}} |T_{\mathbf{k}\mathbf{p}}|^2 \frac{|\Delta_{\mathbf{k}\alpha\alpha}| |\Delta_{\mathbf{p}\alpha\alpha}|}{E_{\mathbf{k}\alpha} E_{\mathbf{p}\alpha}} F_{\mathbf{k}\mathbf{p}\alpha\alpha} \\ &\quad \times [\sin \Delta\theta_{\alpha\alpha} \cos \Delta\theta_{\mathbf{p}\mathbf{k}} + \cos \Delta\theta_{\alpha\alpha} \sin \Delta\theta_{\mathbf{p}\mathbf{k}}] \end{aligned} \quad (35)$$

with $\Delta\theta_{\alpha\alpha} \equiv \theta_{\alpha\alpha}^L - \theta_{\alpha\alpha}^R$, and $\Delta_{\mathbf{k}\alpha\alpha}$ is the surviving order parameter. As expected, the spin-current changes sign depending on whether it is the $\Delta_{\mathbf{k}\uparrow\uparrow}$ or $\Delta_{\mathbf{k}\downarrow\downarrow}$ order parameter that is present.

For collinear magnetization ($\vartheta = 0$), an ordinary Josephson effect occurs with the superconducting phase difference as the driving force. Interestingly, one is able to tune both the spin- and charge-current to zero in the A1-phase when $\mathbf{m}_L \parallel -\mathbf{m}_R$ ($\vartheta = \pi$). It follows from Eq. (34) that the spin- and charge-current only differ by a constant pre-factor

$$I_{tp}^C / I_{tp,z}^S = -\alpha e, \quad \alpha = \pm 1. \quad (36)$$

It is then reasonable to draw the conclusion that we are dealing with a completely *spin-polarized current* such that both I_{ip}^C and $I_{\text{ip},z}^S$ must vanish simultaneously at $\vartheta = \pi$.

Another result that can be extracted from Eqs. (32) and (34) is a persistent non-zero DC spin-Josephson current even if the magnetizations on each side of the junction are of equal magnitude and collinear ($\vartheta = 0$). This is quite different from the spin-Josephson effect recently considered in ferromagnetic metal junctions²⁴. In that case, a twist in the magnetization across the junction is required to drive the spin-Josephson effect.

Note that in the common approximation $T_{\mathbf{k}\mathbf{p}} = T$, *i.e.* the tunneling probability is independent of the electron magnitude and direction of electron momentum, the two-particle current predicted above is identically equal to zero. Of course, such a crude approximation does not correspond to the correct physical picture (see *e.g.* Ref. 51), and in general one cannot neglect the directional dependence of the tunneling matrix element. This demonstrates that we are dealing with a more subtle effect than what could be unveiled when applying the approximation of a constant tunneling matrix element.

An interesting situation arises in the case of zero externally applied voltage *and* identical superconductors on each side of the junction with SC phase differences $\Delta\theta_{\sigma\sigma} = 0$. In this case, we find that $I_J^C = 0$ while

$$I_{J,z}^S = -2 \sum_{\mathbf{k}\mathbf{p}} |T_{\mathbf{k}\mathbf{p}}|^2 \sin^2(\vartheta/2) |\Delta_{\mathbf{k}\uparrow\uparrow} \Delta_{\mathbf{p}\downarrow\downarrow}| F_{\mathbf{k}\mathbf{p}\uparrow\downarrow} \times \sin(\theta_{\downarrow}^L - \theta_{\uparrow}^R) / (E_{\mathbf{k}\uparrow} E_{\mathbf{p}\downarrow}). \quad (37)$$

when $eV = 0$, $\Delta\theta_{\sigma\sigma} = 0$. Thus, we have found a dissipationless spin-current in the two-particle channel without an externally applied voltage *and* without a SC phase difference. This effect is present as long as ϑ is not 0 or π , corresponding to parallel or anti-parallel magnetization on each side of the junction. It is seen from Eq. (37) that the spin-current is driven by an interband phase difference on each side of the junction. A necessary condition for this effect to occur is that no inter-band Josephson coupling is present, *i.e.* electrons in the two energy-bands $E_{\mathbf{k}\uparrow}$ and $E_{\mathbf{k}\downarrow}$ do not communicate with each other. To understand why a Josephson coupling would

destroy the above effect, consider the free energy density for a p -wave FMSC first proposed in Ref. 10, given by

$$\mathcal{F} = \mathcal{F}' - \lambda_J \cos(\theta_{\uparrow\uparrow} - \theta_{\downarrow\downarrow}) \quad (38)$$

in the presence of a Josephson coupling. In Eq. (38), λ_J determines the strength of the interaction while \mathcal{F}' contains the SC and FM contribution to the free energy density in addition to the coupling terms between the SC and FM order parameters. Consequently, the phase difference $\theta_{\uparrow\uparrow} - \theta_{\downarrow\downarrow}$ is locked to 0 or π in order to minimize \mathcal{F} , depending on $\text{sgn}(\lambda_J)$. Considering Eq. (37), we see that $I_{J,z}^S = 0$ in this case, since the argument of the last sine is zero. Mechanisms that would induce a Josephson coupling include magnetic impurities causing inelastic spin-flip scattering between the energy-bands and spin-orbit coupling. Recently, the authors of Ref. 50 proposed that p -wave SC arising out of a FM metal state could be explained by the Berry curvature field that is present in ferromagnets with spin-orbit coupling. It is clear that in the case where spin-orbit coupling is included in the problem, spin-flip scattering processes occur between the energy bands such that the \uparrow and \downarrow spins can not be considered as two independent species any more. The SC phases will then be locked to each other with a relative phase of 0 or π . However, note that in the general case, Eq. (32) produces a non-zero charge- and spin-current even if the spin-up and spin-down phases are locked to each other.

E. Single-particle currents

In the single-particle channel, we find that the charge and spin-currents read

$$I_{\text{sp}}^C = -e \sum_{\alpha} \langle \dot{N}_{\alpha\alpha}(t) \rangle_{\text{sp}} \\ I_{\text{sp},z}^S = \sum_{\alpha} \alpha \langle \dot{N}_{\alpha\alpha}(t) \rangle_{\text{sp}}, \quad (39)$$

as seen from Eq. (29). From Eq. (A1), we then extract the proper expectation value, which is found to be

$$\begin{aligned} \langle \dot{N}_{\alpha\alpha}(t) \rangle_{\text{sp}} = & 4\pi \sum_{\mathbf{k}\mathbf{p}\sigma} [1 + \sigma\alpha \cos(\vartheta)] |T_{\mathbf{k}\mathbf{p}\alpha}|^2 N_{\mathbf{k}\alpha}^2 N_{\mathbf{p}\sigma}^2 \\ & \times \left[[f(E_{\mathbf{k}\alpha}) - f(E_{\mathbf{p}\sigma})] \left(\delta(-eV + E_{\mathbf{k}\alpha} - E_{\mathbf{p}\sigma}) - \frac{|\Delta_{\mathbf{k}\alpha\alpha} \Delta_{\mathbf{p}\sigma\sigma}|^2}{(\xi_{\mathbf{k}\alpha} + E_{\mathbf{k}\alpha})^2 (\xi_{\mathbf{p}\sigma} + E_{\mathbf{p}\sigma})^2} \delta(-eV - E_{\mathbf{p}\sigma} + E_{\mathbf{k}\alpha}) \right) \right. \\ & \left. + [1 - f(E_{\mathbf{k}\alpha}) - f(E_{\mathbf{p}\sigma})] \left(\frac{|\Delta_{\mathbf{k}\alpha\alpha}|^2}{(\xi_{\mathbf{k}\alpha} + E_{\mathbf{k}\alpha})^2} \delta(-eV - E_{\mathbf{k}\alpha} - E_{\mathbf{p}\sigma}) - \frac{|\Delta_{\mathbf{p}\sigma\sigma}|^2}{(\xi_{\mathbf{p}\sigma} + E_{\mathbf{p}\sigma})^2} \delta(-eV + E_{\mathbf{k}\alpha} + E_{\mathbf{p}\sigma}) \right) \right]. \end{aligned} \quad (40)$$

The currents in Eq. (39) are thus seen to require an applied voltage in order to flow in the tunneling junction. Clearly,

this is because the Cooper pairs need to be split up in order

for a single-particle current to exist, such that both spin- and charge-currents vanish at $eV = 0$.

In Ref. 24, the presence of a persistent spin-current in the single-particle channel for FM/FM junctions with a twist in magnetization across the junction was predicted. For consistency, our results must confirm this prediction for the single-particle current in the limit where SC is lost, *i.e.* $\Delta_{\mathbf{k}\sigma\sigma} \rightarrow 0$. Note that the \hat{z} -direction in Ref. 24 corresponds to a vector in our local xy -plane since the present quantization axis lies parallel with the magnetization direction. Upon calculating the x - and y -components of the single-particle spin-current for our system in the limit where SC is lost, *i.e.* $\Delta_{\mathbf{k}\sigma\sigma} \rightarrow 0$, a persistent spin Josephson-like current proportional to $\sin(\vartheta)$ is identified. More precisely,

$$\mathbf{I}_{\text{sp}}^S(t) = 2 \sum_{\mathbf{k}\mathbf{p}} \sum_{\alpha\beta\sigma} \hat{\mathcal{D}}_{\sigma\alpha}^{(1/2)}(\vartheta) \hat{\mathcal{D}}_{\sigma\beta}^{(1/2)}(\vartheta) |T_{\mathbf{k}\mathbf{p}}|^2 \times \text{Im}\{\hat{\sigma}_{\beta\alpha} \Lambda_{\beta\sigma}^{1,1}(-eV)\} \quad (41)$$

when $\Delta_{\mathbf{q}\sigma\sigma} = 0$ (see Appendix for details). In agreement with Ref. 24, the component of the spin-current parallel to $\mathbf{m}_L \times \mathbf{m}_R$ is seen to vanish for $\vartheta = \{0, \pi\}$ at $eV = 0$.

III. FERROMAGNETS WITH SPIN-ORBIT COUPLING

A. Coexistence of ferromagnetism and spin-orbit coupling

In a system where time-reversal and spatial inversion symmetry are simultaneously broken, it is clear that spins are heavily affected by these properties. There is currently much focus on ferromagnetic semiconductors where spin-orbit coupling plays a crucial role with regard to transport properties^{21,22}. In fact, there has in recent years been much progress in the semiconductor research community where the spin-Hall effect in particular has received much attention²³. With the discovery⁵² of hole-mediated ferromagnetic order in (In,Mn)As, extensive research on III-V host materials was triggered. Moreover, it is clear that properties such as ferromagnetic transition temperatures in excess of 100 K⁵³ and long spin-coherence times⁵⁴ in GaAs have strongly contributed to opening up a vista plethora for information processing and storage technologies in these new magnetic mediums⁵⁵.

Generally, spin-orbit coupling (SOC) can be roughly divided into two categories – *intrinsic* and *extrinsic*. Intrinsic SOC is found in materials with a non-centrosymmetric crystal symmetry, *i.e.* where inversion symmetry is broken, whereas extrinsic SOC is due to asymmetries caused by impurities, local confinements of electrons or externally applied electrical fields.

In the present paper, we investigate the tunneling current of spin between two ferromagnetic metals with spin-orbit coupling induced by an external electric field. This way, we will have two externally controllable parameters; the magnetization \mathbf{m} and the electrical field \mathbf{E} . The case of tunneling between two noncentrosymmetric superconductors with signif-

icant spin-orbit coupling, but no ferromagnetism, has previously been considered in Ref. 56.

B. The Hamiltonian

Our system consists of two Heisenberg ferromagnets with substantial spin-orbit coupling, separated by a thin insulating barrier which is assumed to be spin-inactive. This is shown in Fig. 4. We now operate with only one quantization axis, such that a proper tunneling Hamiltonian for this purpose is

$$H_T = \sum_{\mathbf{k}\mathbf{p}\sigma} (T_{\mathbf{k}\mathbf{p}} c_{\mathbf{k}\sigma}^\dagger d_{\mathbf{p}\sigma} + \text{h.c.}), \quad (42)$$

where $\{c_{\mathbf{k}\sigma}^\dagger, c_{\mathbf{k}\sigma}\}$ and $\{d_{\mathbf{k}\sigma}^\dagger, d_{\mathbf{k}\sigma}\}$ are creation and annihilation operators for an electron with momentum \mathbf{k} and spin σ on the right and left side of the junction, respectively, while $T_{\mathbf{k}\mathbf{p}}$ is the spin-independent tunneling matrix element. In \mathbf{k} -

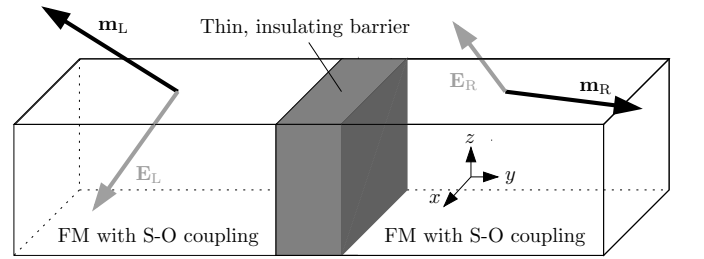


FIG. 4: Our model consisting of two ferromagnetic metals with spin-orbit coupling separated by a thin insulating barrier. The magnetization \mathbf{m} and electrical field \mathbf{E} are allowed to point in any direction so that our results are generally valid, while special cases such as planar magnetization etc. are easily obtained by applying the proper limits to the general expressions.

space, the Hamiltonian describing the ferromagnetism reads

$$H_{\text{FM}} = \sum_{\mathbf{k}\sigma} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} - JN \sum_{\mathbf{k}} \eta(\mathbf{k}) \mathbf{S}_{\mathbf{k}} \cdot \mathbf{S}_{-\mathbf{k}} \quad (43)$$

in which $\varepsilon_{\mathbf{k}}$ is the kinetic energy of the electrons, J is the ferromagnetic coupling constant, N is the number of particles in the system, while $\mathbf{S}_{\mathbf{k}} = (1/2) \sum_{\alpha\beta} c_{\mathbf{k}\alpha}^\dagger \hat{\sigma}_{\alpha\beta} c_{\mathbf{k}\beta}$ is the spin operator. As we later adopt the mean-field approximation, $\mathbf{m} = (m_x, m_y, m_z)$ will denote the magnetization of the system.

The spin-orbit interactions are accounted for by a Rashba Hamiltonian

$$H_{\text{S-O}} = - \sum_{\mathbf{k}} \varphi_{\mathbf{k}}^\dagger [\xi(\nabla V \times \mathbf{k}) \cdot \hat{\sigma}] \varphi_{\mathbf{k}}, \quad (44)$$

where $\varphi_{\mathbf{k}} = [c_{\mathbf{k}\uparrow}, c_{\mathbf{k}\downarrow}]^T$, $\mathbf{E} = -\nabla V$ is the electrical field felt by the electrons and $\hat{\sigma} = (\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$ in which $\hat{\sigma}_i$ are Pauli matrices, while the parameter ξ is material-dependent. From now on, the notation $\xi(\mathbf{E} \times \mathbf{k}) \equiv \mathbf{B}_{\mathbf{k}} = (B_{\mathbf{k},x}, B_{\mathbf{k},y}, B_{\mathbf{k},z})$ will be used. In general, the electromagnetic potential V consists of two parts V_{int} and V_{ext} (see *e.g.* Ref. 23 for a detailed

discussion of the spin-orbit Hamiltonian). The crystal potential of the material is represented by V_{int} , and only gives rise to a spin-orbit coupling if inversion symmetry is broken in the crystal structure. Asymmetries such as impurities and local confinements of electrons are included in V_{ext} , as well as any external electrical field. Note that any lack of crystal inversion symmetry results in a so-called Dresselhaus term in the Hamiltonian, which is present in the absence of any impurities and confinement potentials. In the following, we focus on the spin-orbit coupling resulting from V_{ext} , thus considering any symmetry-breaking electrical field that arises from charged impurities or which is applied externally. In the case where the crystal structure does not respect inversion symmetry, a Dresselhaus term⁵⁷ can be easily included in the Hamiltonian by performing the substitution

$$(\mathbf{E} \times \mathbf{k}) \cdot \hat{\sigma} \rightarrow [(\mathbf{E} \times \mathbf{k}) + \mathcal{D}(\mathbf{k})] \cdot \hat{\sigma}, \quad (45)$$

where $\mathcal{D}(\mathbf{k}) = -\mathcal{D}(-\mathbf{k})$.

We now proceed to calculate the spin-current that is generated across the junction as a result of tunneling. Note that in our model, the magnetization vector and electrical field are allowed to point in arbitrary directions. In this way, the obtained result for the spin-current will be generally valid and special cases, *e.g.* thin films, are easily obtained by taking the appropriate limits in the final result. It should be mentioned that the effective magnetic field from the spin-orbit interactions might influence the direction of the magnetization in the ferromagnet. This is, however, not the main focus of our work, and we leave this question open for study. Our emphasis in the present paper concerns the derivation of general results onto which specific restrictions may be applied as they seem appropriate.

In the mean-field approximation, the Hamiltonian for the right side of the junction can be written as $H = H_{\text{FM}} + H_{\text{S-O}}$, which in a compact form yields

$$H_{\text{R}} = H_0 + \sum_{\mathbf{k}} \varphi_{\mathbf{k}}^{\dagger} \begin{pmatrix} \varepsilon_{\mathbf{k}\uparrow} & -\zeta_{\text{R}} + B_{\mathbf{k},-} \\ -\zeta_{\text{R}}^{\dagger} + B_{\mathbf{k},+} & \varepsilon_{\mathbf{k}\downarrow} \end{pmatrix} \varphi_{\mathbf{k}}, \quad (46)$$

where $\varepsilon_{\mathbf{k}\sigma} \equiv \varepsilon_{\mathbf{k}} - \sigma(\zeta_{\text{R}} - B_{\mathbf{k},z})$ and H_0 is an irrelevant constant. The FM order parameters are $\zeta_{\text{R}} = 2J\eta(0)(m_{\text{R},x} - im_{\text{R},y})$ and $\zeta_{\text{z,R}} = 2J\eta(0)m_{\text{R},z}$ and $B_{\mathbf{k},\pm} \equiv B_{\mathbf{k},x} \pm iB_{\mathbf{k},y}$. For convenience, we from now on write $\zeta = |\zeta|e^{i\phi}$ and $B_{\mathbf{k},\pm} = |B_{\mathbf{k},\pm}|e^{\mp i\chi_{\mathbf{k}}}$. The Hamiltonian for the left side of the junction is obtained from Eq. (46) simply by the doing the replacements $\mathbf{k} \rightarrow \mathbf{p}$ and $\text{R} \rightarrow \text{L}$.

C. Tunneling formalism

In order to obtain the expressions for the spin- and charge-tunneling currents, it is necessary to calculate the Green functions. These are given by the matrix

$$\hat{G}_{\mathbf{k}}(i\omega_n) = (-i\omega_n \hat{1} + \hat{A}_{\mathbf{k}})^{-1}, \quad (47)$$

where $\hat{A}_{\mathbf{k}}$ is the matrix in Eq. (46). Explicitly, we have that

$$\hat{G}_{\mathbf{k}}(i\omega_n) = \begin{pmatrix} G_{\mathbf{k}}^{\uparrow\uparrow}(i\omega_n) & F_{\mathbf{k}}^{\downarrow\uparrow}(i\omega_n) \\ F_{\mathbf{k}}^{\uparrow\downarrow}(i\omega_n) & G_{\mathbf{k}}^{\downarrow\downarrow}(i\omega_n) \end{pmatrix}. \quad (48)$$

Above, $\omega_n = 2(n+1)\pi/\beta$, $n = 0, 1, 2, \dots$ is the fermionic Matsubara frequency and β denotes inverse temperature. Introducing

$$X_{\mathbf{k}}(i\omega_n) = (\varepsilon_{\mathbf{k}\uparrow} - i\omega_n)(\varepsilon_{\mathbf{k}\downarrow} - i\omega_n) - |\zeta_{\text{R}} - B_{\mathbf{k},-}|^2, \quad (49)$$

the normal and anomalous Green functions are

$$G_{\mathbf{k}}^{\sigma\sigma}(i\omega_n) = (\varepsilon_{\mathbf{k},-\sigma} - i\omega_n)/X_{\mathbf{k}}(i\omega_n), \\ F_{\mathbf{k}}^{\uparrow\downarrow}(i\omega_n) = F_{\mathbf{k}}^{\downarrow\uparrow}(i\omega_n) = (\zeta_{\text{R}} - B_{\mathbf{k},-})/X_{\mathbf{k}}(i\omega_n). \quad (50)$$

The expression for $\mathbf{I}^{\text{S}}(t)$ is established by first considering the generalized number operator $N_{\alpha\beta} = \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{\dagger} c_{\mathbf{k}\beta}$. This operator changes with time due to tunneling according to $\dot{N}_{\alpha\beta} = i[H_{\text{T}}, N_{\alpha\beta}]$, which in the interaction picture representation becomes $\dot{N}_{\alpha\beta}(t) = -i \sum_{\mathbf{k}\mathbf{p}} (T_{\mathbf{k}\mathbf{p}} c_{\mathbf{k}\alpha}^{\dagger} d_{\mathbf{p}\beta} e^{iteV} - \text{h.c.})$. The voltage drop across the junction is given by the difference in chemical potential on each side, *i.e.* $eV = \mu_{\text{R}} - \mu_{\text{L}}$. In the linear response regime, the spin-current across the junction is

$$\mathbf{I}^{\text{S}}(t) = \frac{1}{2} \sum_{\alpha\beta} \hat{\sigma}_{\alpha\beta} \langle \dot{N}_{\alpha\beta}(t) \rangle, \quad (51)$$

where the expectation value of the time derivative of the transport operator is calculated by means of the Kubo formula Eq. (30). Details will be given in Sec. A 2.

D. Single-particle currents

At $eV = 0$, it is readily seen from the discussion in Sec. A 2 that the charge-current vanishes. Consider now the z -component of the spin-current in particular, which can be written as $I_z^{\text{S}} = \Im\{\Phi(-eV)\}$. The Matsubara function $\Phi(-eV)$ is found by performing analytical continuation $i\tilde{\omega}_{\nu} \rightarrow -eV + i0^{+}$ on $\tilde{\Phi}(i\tilde{\omega}_{\nu})$, where

$$\tilde{\Phi}(i\tilde{\omega}_{\nu}) = \frac{1}{\beta} \sum_{i\omega_m, \mathbf{k}\mathbf{p}} \sum_{\sigma} \sigma \left(G_{\mathbf{k}}^{\sigma\sigma}(i\omega_m) G_{\mathbf{p}}^{\sigma\sigma}(i\omega_m - i\tilde{\omega}_{\nu}) \right. \\ \left. + F_{\mathbf{k}}^{-\sigma, \sigma}(i\omega_m) F_{\mathbf{p}}^{\sigma, -\sigma}(i\omega_m - i\tilde{\omega}_{\nu}) \right). \quad (52)$$

Here, $\tilde{\omega}_{\nu} = 2\nu\pi/\beta$, $\nu = 0, 1, 2, \dots$ is the bosonic Matsubara frequency. Inserting the Green functions from Eq. (50) into Eq. (52), one finds that a persistent spin-current is established across the tunneling junction. For zero applied voltage, we obtain

$$I_z^{\text{S}} = \sum_{\mathbf{k}\mathbf{p}} \frac{|T_{\mathbf{k}\mathbf{p}}|^2 J_{\mathbf{k}\mathbf{p}}}{2\gamma_{\mathbf{k}}\gamma_{\mathbf{p}}} \left[|\zeta_{\text{R}}\zeta_{\text{L}}| \sin \Delta\phi + |B_{\mathbf{k},-}B_{\mathbf{p},-}| \sin \Delta\chi_{\mathbf{k}\mathbf{p}} \right. \\ \left. - |B_{\mathbf{k},-\zeta_{\text{L}}}| \sin(\chi_{\mathbf{k}} - \phi_{\text{L}}) - |B_{\mathbf{p},-\zeta_{\text{R}}}| \sin(\phi_{\text{R}} - \chi_{\mathbf{p}}) \right], \quad (53a)$$

$$J_{\mathbf{k}\mathbf{p}} = \sum_{\substack{\alpha=\pm \\ \beta=\pm}} \alpha\beta \left[\frac{n(\varepsilon_{\mathbf{k}} + \alpha\gamma_{\mathbf{k}}) - n(\varepsilon_{\mathbf{p}} + \beta\gamma_{\mathbf{p}})}{(\varepsilon_{\mathbf{k}} + \alpha\gamma_{\mathbf{k}}) - (\varepsilon_{\mathbf{p}} + \beta\gamma_{\mathbf{p}})} \right]. \quad (53b)$$

In Eqs. (53), $\Delta\chi_{\mathbf{k}\mathbf{p}} \equiv \chi_{\mathbf{k}} - \chi_{\mathbf{p}}$, $\Delta\phi \equiv \phi_{\mathbf{R}} - \phi_{\mathbf{L}}$, while

$$\gamma_{\mathbf{k}}^2 = (\zeta_{z,\mathbf{R}} - B_{\mathbf{k},z})^2 + |\zeta_{\mathbf{R}} - B_{\mathbf{k},-}|^2 \quad (54)$$

and $n(\varepsilon)$ denotes the Fermi distribution. In the above expressions, we have implicitly associated the right side R with the momentum label \mathbf{k} and L with \mathbf{p} for more concise notation, such that *e.g.* $B_{\mathbf{k},z} \equiv B_{\mathbf{k},z}^{\mathbf{R}}$. Defining $\zeta_i = 2J\eta(0)m_i$, we see that Eq. (54) can be written as

$$\gamma_{\mathbf{k}} = |\zeta_{\mathbf{R}} - \mathbf{B}_{\mathbf{k}}|. \quad (55)$$

The spin-current described in Eq. (53) can be controlled by adjusting the relative orientation of the magnetization vectors on each side of the junction, *i.e.* $\Delta\phi$, and also responds to a change in direction of the applied electric fields. The presence of an external magnetic field \mathbf{H}_i would control the orientation of the internal magnetization \mathbf{m}_i . Alternatively, one may also use exchange biasing to an anti-ferromagnet in order to lock the magnetization direction. Consequently, the spin-current can be manipulated by the external control parameters $\{\mathbf{H}_i, \mathbf{E}_i\}$ in a well-defined manner. This observation is highly suggestive in terms of novel nanotechnological devices.

We stress that Eq. (53) is *non-zero* in the general case, since $\gamma_{\mathbf{k}} \neq -\gamma_{-\mathbf{k}}$ and $\chi_{-\mathbf{k}} = \chi_{\mathbf{k}} + \pi$. Moreover, Eq. (53) is valid for any orientation of both \mathbf{m} and \mathbf{E} on each side of the junction, and a number of interesting special cases can now easily be considered simply by applying the appropriate limits to this general expression.

E. Special limits

Consider first the limit where ferromagnetism is absent, such that the tunneling occurs between two bulk materials with spin-orbit coupling. Applying $\mathbf{m} \rightarrow 0$ to Eq. (53), it is readily seen that the spin-current vanishes for any orientation of the electrical fields. Intuitively, one can understand this by considering the band structure of the quasi-particles with energy $E_{\mathbf{k}\sigma} = \varepsilon_{\mathbf{k}} + \sigma\gamma_{\mathbf{k}}$ and the corresponding density of states $N(E_{\mathbf{k}\sigma})$ when only spin-orbit coupling is present, as shown in Fig. 5. Since the density of states is equal for \uparrow and \downarrow spins⁶⁹, one type of spin is not preferred compared to the other with regard to tunneling, resulting in a net spin-current of zero. Formally, the vanishing of the spin-current can be understood by replacing the momentum summation with integration over energy, *i.e.* $\sum_{\mathbf{k}\mathbf{p}} \rightarrow \int \int dE_{\mathbf{R}} dE_{\mathbf{L}} N_{\mathbf{R}}(E_{\mathbf{R}}) N_{\mathbf{L}}(E_{\mathbf{L}})$. When $\mathbf{m} \rightarrow 0$, Eq. (53) dictates that

$$I_z^S \sim \sum_{\substack{\alpha=\pm \\ \beta=\pm}} \alpha\beta \int \int dE_{\mathbf{R},\alpha} dE_{\mathbf{L},\beta} N_{\mathbf{R}}^{\alpha}(E_{\mathbf{R},\alpha}) N_{\mathbf{L}}^{\beta}(E_{\mathbf{L},\beta}) \times \left[\frac{n(E_{\mathbf{R},\alpha}) - n(E_{\mathbf{L},\beta})}{E_{\mathbf{R},\alpha} - E_{\mathbf{L},\beta}} \right]. \quad (56)$$

Since the density of states for the \uparrow - and \downarrow -populations are equal in the individual subsystems, *i.e.* $N^{\uparrow}(E) = N^{\downarrow}(E) \equiv N(E)$, the integrand of Eq. (56) becomes spin-independent

such that the summation over α and β yields zero. Thus, no spin-current will exist at $eV = 0$ over a tunneling barrier separating two systems with spin-orbit coupling alone. In the general case where both ferromagnetism and spin-orbit coupling are present, the density of states at, say, Fermi level are different, leading to a persistent spin-current across the junction due to the difference between $N^{\uparrow}(E)$ and $N^{\downarrow}(E)$.

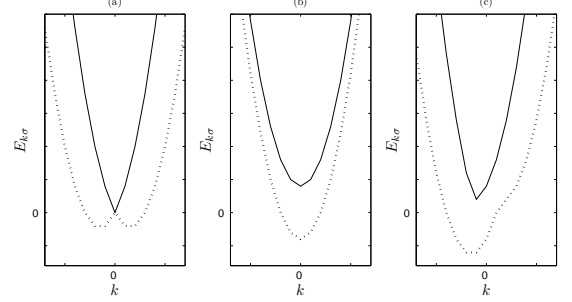


FIG. 5: Schematic illustration of the energy-bands for (a) a system with spin-orbit coupling, (b) a system with ferromagnetic ordering, and (c) a system exhibiting both of the aforementioned properties. The dotted line corresponds to quasi-particles with $\sigma = \downarrow$, while the full drawn line designates $\sigma = \uparrow$. Since the density of states $N^{\sigma}(E_{k\sigma})$ is proportional to $(\partial E_{k\sigma}/\partial k)^{-1}$, we see that a difference between $N^{\uparrow}(E_{k\sigma})$ and $N^{\downarrow}(E_{k\sigma})$ is zero at Fermi level in (a), while the density of states differ for the \uparrow - and \downarrow -populations in (b) and (c). Thus, a persistent spin-current will only occur for tunneling between systems corresponding to (b) and (c).

We now consider a special case where the bulk structures indicated in Fig. 4 are reduced to two thin-film ferromagnets in the presence of electrical fields that are perpendicular to each other, say $\mathbf{E}_{\mathbf{L}} = (E_{\mathbf{L}}, 0, 0)$ and $\mathbf{E}_{\mathbf{R}} = (0, E_{\mathbf{R}}, 0)$, as shown in Fig. 6(a) and (b). In this case, we have chosen an in-plane magnetization for each of the thin-films. Solving specifically for Fig. 6(a), it is seen that $\mathbf{m}_{\mathbf{L}} = (0, m_{\mathbf{L},y}, m_{\mathbf{L},z})$ and $\mathbf{m}_{\mathbf{R}} = (m_{\mathbf{R},x}, 0, m_{\mathbf{R},z})$. Furthermore, assume that the electrons are restricted from moving in the “thin” dimension, *i.e.* $\mathbf{p} = (0, p_y, p_z)$ and $\mathbf{k} = (k_x, 0, k_z)$. In this case, Eq. (53) reduces to the form

$$I_z^S = I_0 \text{sgn}(m_{\mathbf{L},y}) + \sum_{\mathbf{k}\mathbf{p}} I_{1,\mathbf{k}\mathbf{p}} \text{sgn}(p_z), \quad (57)$$

where the constants above are

$$I_0 = \sum_{\mathbf{k}\mathbf{p}} \frac{|T_{\mathbf{k}\mathbf{p}}|^2 J_{\mathbf{k}\mathbf{p}} (|\zeta_{\mathbf{R}} \zeta_{\mathbf{L}}| - E_{\mathbf{R}} |k_z \zeta_{\mathbf{L}}|)}{2|\zeta_{\mathbf{R}} + \mathbf{B}_{\mathbf{k}}||\zeta_{\mathbf{L}} + \mathbf{B}_{\mathbf{p}}|}, \quad (58)$$

$$I_{1,\mathbf{k}\mathbf{p}} = \frac{|T_{\mathbf{k}\mathbf{p}}|^2 J_{\mathbf{k}\mathbf{p}} E_{\mathbf{L}} (E_{\mathbf{R}} |k_z p_z| - |p_z \zeta_{\mathbf{R}}|)}{2|\zeta_{\mathbf{R}} + \mathbf{B}_{\mathbf{k}}||\zeta_{\mathbf{L}} + \mathbf{B}_{\mathbf{p}}|},$$

with

$$\begin{aligned} \zeta_{\mathbf{L}} &= 2J\eta(0)(0, m_{\mathbf{L},y}, m_{\mathbf{L},z}) & \mathbf{B}_{\mathbf{p}} &= \xi_{\mathbf{L}} E_{\mathbf{L}}(0, -p_z, p_y) \\ \zeta_{\mathbf{R}} &= 2J\eta(0)(m_{\mathbf{R},x}, 0, m_{\mathbf{R},z}) & \mathbf{B}_{\mathbf{k}} &= \xi_{\mathbf{R}} E_{\mathbf{R}}(k_y, 0, -k_x) \end{aligned} \quad (59)$$

such that $I_{1,\mathbf{k}\mathbf{p}} \neq I_{1,-\mathbf{k},-\mathbf{p}}$. Likewise, for the setup sketched in Fig. 6(b), one obtains

$$I_z^S = \sum_{\mathbf{k}\mathbf{p}} \frac{|T_{\mathbf{k}\mathbf{p}}|^2 J_{\mathbf{k}\mathbf{p}}}{2|\zeta_R + E_R(k_y - ip_x)|^2 |\zeta_L + E_L(p_y - ip_x)|^2} \times \left[|\zeta_R \zeta_L| \sin \Delta\phi + E_R E_L (k_y^2 + k_x^2)(p_y^2 + p_x^2) |\sin \Delta\chi_{\mathbf{k}\mathbf{p}} - E_R |\zeta_L| (k_y^2 + k_x^2) \sin(\chi_k - \phi_L) - E_L |\zeta_R| (p_y^2 + p_x^2) \sin(\phi_R - \chi_{\mathbf{p}}) \right], \quad (60)$$

where $\chi_{\mathbf{q}}$ obeys

$$\tan \chi_{\mathbf{q}} = -\frac{q_x}{q_y}, \quad \mathbf{q} = \mathbf{k}, \mathbf{p}. \quad (61)$$

From these observations, we can draw the following conclusions: whereas the spin-current is zero for the system in Fig. 6(a) and (b) if only spin-orbit coupling is considered, it is non-zero when only ferromagnetism is taken into account. However, in the general case where both ferromagnetism and spin-orbit coupling are included, an *additional term* in the spin-current is induced compared to the pure ferromagnetic case. Accordingly, there is an interplay between the magnetic order and the Rashba-interaction that produces a spin-current which is more than just the sum of the individual contributions.

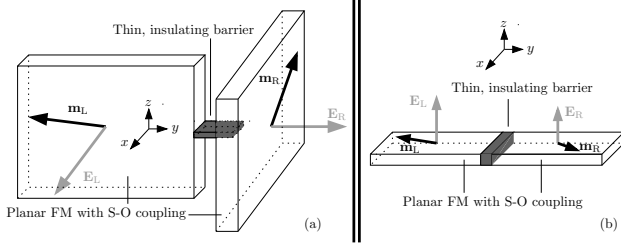


FIG. 6: Tunneling between planar ferromagnets in the presence of externally applied electrical fields \mathbf{E}_L and \mathbf{E}_R that destroy inversion symmetry and induce a spin-orbit coupling.

IV. DISCUSSION OF RESULTS

Having presented the general results for tunneling currents between systems with multiple broken symmetries in the preceding sections, we now focus on detection and experimental issues concerning verification of our predictions.

Consider first the system consisting of two ferromagnetic superconductors separated by a thin, insulating barrier. It is well-known that for tunneling currents flowing between two s -wave SC in the presence of a magnetic field that is perpendicular to the tunneling direction, the resulting flux threading the junction leads to a Fraunhofer-like variation in the DC Josephson effect, given by a multiplicative factor

$$D_F(\Phi) = \frac{\sin(\pi\Phi/\Phi_0)}{(\pi\Phi/\Phi_0)} \quad (62)$$

in the critical current. Here, $\Phi_0 = \pi\hbar/e$ is the elementary flux quantum, and Φ is the total flux threading the junction due to a magnetic field. Consequently, the presence of magnetic flux in the tunneling junction of two s -wave SC threatens to nullify the total Josephson current. In the present case of two p -wave FMSC, this is not an issue since we have assumed uniform coexistence of the SC and FM order parameters which is plausible for a weak intrinsic magnetization. The effect of an external magnetic field \mathbf{H} would then simply be to rotate the internal magnetization as dictated by the term $-\mathbf{H} \cdot \mathbf{m}$ in the free energy \mathcal{F} (see *e.g.* Ref. 18). Thus, there is no diffraction pattern present for the tunneling-currents between two non-unitary ESP FMSC, regardless of how the internal magnetization is oriented. Since the motion of the Cooper-pairs is also restricted by the thin-film structure, there is no orbital effect from such a magnetization.

Note that the interplay between ferromagnetism and superconductivity is manifest in the charge- as well as spin-currents, the former being readily measurable. Detection of the induced spin-currents would be challenging, although recent studies suggest feasible methods of measuring such quantities⁵⁸. We comment more on this later in this section. First, we address the issue of how boundary effects affect the order parameters. Studies^{40,41,42} have shown that interfaces/surfaces may have a pair-breaking effect on unconventional SC order parameters. This is highly relevant in tunneling junction experiments as in the present case. The suppression of the order parameter is caused by a formation of so-called midgap surface states (also known as zero-energy states)³⁹ which occurs for certain orientations of the \mathbf{k} -dependent SC gaps that satisfy a resonance condition. Note that this is not the case for conventional s -wave superconductors since the gap is isotropic in that case. This pair-breaking surface effect was studied specifically for p -wave order parameters in Refs. 40,41, and it was found that the component of the order parameter that experiences a sign change under the transformation $k_{\perp} \rightarrow -k_{\perp}$, where k_{\perp} is the component of momentum perpendicular to the tunneling junction, was suppressed in the vicinity of the junction. By vicinity of the junction, we here mean a distance comparable to the coherence length, typically of order 1-10 nm. Thus, depending on the explicit form of the superconducting gaps in the FMSC, these could be subject to a reduction close to the junction, which in turn would reduce the magnitude of the Josephson effect we predict. Nevertheless, the latter is nonvanishing in the general case.

Since the critical Josephson currents depend on the relative magnetization orientation, one is able to tune these currents in a well-defined manner by varying ϑ . This can be done by applying an external magnetic field in the plane of the FMSC. In the presence of a rotating magnetic moment on either side of the junction, the Josephson currents will thus vary according to Eq. (32), which may be cast into the form $I_J^C = I_0 + I_m \cos(\vartheta)$. Depending on the relative magnitudes of I_0 and I_m , the sign of the critical current may change. Note that such a variation of the magnetization vectors must take place in an adiabatic manner so that the systems can be considered to be in, or near, equilibrium at all

times. Our predictions can thus be verified by measuring the critical current at $eV = 0$ for different angles ϑ and compare the results with our theory. Recently, it has been reported that a spin-triplet supercurrent, induced by Josephson tunneling between two s -wave superconductors across a ferromagnetic metallic contact, can be controlled by varying the magnetization of the ferromagnetic contact⁵⁹. Moreover, concerning the spin-Josephson current we propose, detection of induced spin-currents are challenging, although recent studies suggest feasible methods of measuring such quantities⁵⁸. Observation of macroscopic spin-currents in superconductors may also be possible via angle resolved photo-emission experiments with circularly polarized photons⁶⁰, or in spin-resolved neutron scattering experiments³².

We reemphasize that the above ideas should be experimentally realizable by *e.g.* utilizing various geometries in order to vary the demagnetization fields. Alternatively, one may use exchange biasing to an anti-ferromagnet. Such techniques of achieving non-collinearity are routinely used in ferromagnet-normal metal structures⁶¹.

With regard to the predicted DC spin-current in for a system consisting of two ferromagnetic metals with spin-orbit coupling, we here suggest how this effect could be probed for in an experimental setup. For instance, the authors of Ref. 62 propose a spin-mechanical device which exploits nanomechanical torque for detection and control of a spin-current. Similarly, a setup coupling the electron spin to the mechanical motion of a nanomechanical system is proposed in Ref. 58. The latter method employs the strain-induced spin-orbit interaction of electrons in a narrow gap semiconductor. In Ref. 63, it was demonstrated that a steady-state magnetic-moment current, *i.e.* spin-current, will induce a static electric field. This fact may be suggestive in terms of detection^{64,65}, and could be useful to observe the novel effects predicted in this paper.

V. SUMMARY

In summary, we have considered supercurrents of spin and charge that exist in FMSC/FMSC and FMSO/FMSO tunneling junctions. In the former case, we have found an interplay between the relative magnetization orientation on each side of the junction and the SC phase difference when considering tunneling between two non-unitary ESP FMSC with coexisting and uniform FM and SC order. This interplay is present in the Josephson channel, offering the opportunity to tune dissipationless currents of spin and charge in a well-defined manner by adjusting the relative magnetization orientation on each side of the junction. As a special case, we considered the case where the SC phase difference is zero, and found that a dissipationless spin-current *without* charge-current would be established across the junction. Suggestions concerning the detection of the effects we predict have been made.

Moreover, we have derived an expression for a dissipationless spin-current that arises in the junction between two Heisenberg ferromagnets with spin-orbit coupling. We have shown that the spin-current is driven by terms originating from both the ferromagnetic phase difference, in agreement with

the result of Ref. 24, and the presence of spin-orbit coupling itself. In addition, it was found that the simultaneous breaking of time-reversal and inversion symmetry fosters an interplay between ferromagnetism and spin-orbit coupling in the spin-current. Availing oneself of external magnetic and electric fields, our expressions show that the spin-current can be tuned in a well-defined manner. These results are of significance in the field of spintronics in terms of quantum transport, and offer insight into how the spin-current behaves for nanostructures exhibiting both ferromagnetism and spin-orbit coupling.

Acknowledgments

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APPENDIX A: DETAILS OF MATSUBARA FORMALISM

1. Ferromagnetic superconductors

Inserting Eq. (27) into Eq. (30), one finds that

$$\begin{aligned} \langle \dot{N}_{\alpha\beta}(t) \rangle &= \langle \dot{N}_{\alpha\beta}(t) \rangle_{\text{sp}} + \langle \dot{N}_{\alpha\beta}(t) \rangle_{\text{tp}} \\ &= - \int_{-\infty}^t dt' \left[\langle [M_{\alpha\beta}(t), M^\dagger(t')] \rangle e^{-ieV(t-t')} \right. \\ &\quad - \langle [M_{\beta\alpha}^\dagger(t), M(t')] \rangle e^{ieV(t-t')} \\ &\quad + \langle [M_{\alpha\beta}(t), M(t')] \rangle e^{-ieV(t+t')} \\ &\quad \left. - \langle [M_{\beta\alpha}^\dagger(t), M^\dagger(t')] \rangle e^{ieV(t+t')} \right] \end{aligned} \quad (\text{A1})$$

where the two first terms in Eq. (A1) contribute to the single-particle current while the two last terms constitute the Josephson current. Above, we defined

$$\begin{aligned} M_{\alpha\beta}(t) &= \sum_{\mathbf{k}\mathbf{p}\sigma} \hat{D}_{\sigma\beta}^{(1/2)}(\vartheta) T_{\mathbf{k}\mathbf{p}} c_{\mathbf{k}\alpha}^\dagger(t) d_{\mathbf{p}\sigma}(t) \\ M(t) &= \sum_{\mathbf{k}\mathbf{p}\sigma\sigma'} \hat{D}_{\sigma'\sigma}^{(1/2)}(\vartheta) T_{\mathbf{k}\mathbf{p}} c_{\mathbf{k}\sigma}^\dagger(t) d_{\mathbf{p}\sigma'}(t). \end{aligned} \quad (\text{A2})$$

By observing that $\hat{\sigma}_{\alpha\beta} = (\hat{\sigma}_{\beta\alpha})^*$, we can combine Eqs. (30)-(A2) to yield

$$\begin{aligned} \hat{\sigma}_{\alpha\beta} \langle \dot{N}_{\alpha\beta}(t) \rangle_{\text{sp}} &= 2\Im\{\hat{\sigma}_{\alpha\beta} \Phi_{\alpha\beta,\text{sp}}(-eV)\} \\ \hat{\sigma}_{\alpha\beta} \langle \dot{N}_{\alpha\beta}(t) \rangle_{\text{tp}} &= 2\Im\{\hat{\sigma}_{\alpha\beta} \Phi_{\alpha\beta,\text{J}}(eV) e^{-2ietV}\} \end{aligned} \quad (\text{A3})$$

where the Matsubara functions are obtained by performing analytical continuation according to

$$\begin{aligned} \Phi_{\alpha\beta,\text{sp}}(-eV) &= \lim_{i\tilde{\omega}_\nu \rightarrow -eV + i0^+} \tilde{\Phi}_{\alpha\beta,\text{sp}}(i\tilde{\omega}_\nu) \\ \Phi_{\alpha\beta,\text{J}}(eV) &= \lim_{i\tilde{\omega}_\nu \rightarrow eV + i0^+} \tilde{\Phi}_{\alpha\beta,\text{tp}}(i\tilde{\omega}_\nu), \end{aligned} \quad (\text{A4})$$

In Eq. (A4), $\tilde{\omega}_\nu = 2\pi\nu/\beta$, $\nu = 1, 2, 3 \dots$ is the bosonic Matsubara frequency and

$$\begin{aligned}\tilde{\Phi}_{\text{sp},\alpha\beta}(1\tilde{\omega}_\nu) &= - \int_0^\beta d\tau e^{i\tilde{\omega}_\nu\tau} \sum_{\substack{\mathbf{k}\mathbf{p}\sigma \\ \mathbf{k}'\mathbf{p}'\sigma_1\sigma_2}} \hat{\mathcal{D}}_{\sigma\beta}^{(1/2)}(\vartheta) \hat{\mathcal{D}}_{\sigma_1\sigma_2}^{(1/2)}(\vartheta) \times \\ &\quad T_{\mathbf{k}\mathbf{p}} T_{\mathbf{k}'\mathbf{p}'}^* \langle \tilde{T} \{ c_{\mathbf{k}\alpha}^\dagger(\tau) d_{\mathbf{p}\sigma}(\tau) d_{\mathbf{p}'\sigma_1}^\dagger(0) c_{\mathbf{k}'\sigma_2}(0) \} \rangle, \\ \tilde{\Phi}_{\text{tp},\alpha\beta}(1\tilde{\omega}_\nu) &= - \int_0^\beta d\tau e^{i\tilde{\omega}_\nu\tau} \sum_{\substack{\mathbf{k}\mathbf{p}\sigma \\ \mathbf{k}'\mathbf{p}'\sigma_1\sigma_2}} \hat{\mathcal{D}}_{\sigma\beta}^{(1/2)}(\vartheta) \hat{\mathcal{D}}_{\sigma_1\sigma_2}^{(1/2)}(\vartheta) \times \\ &\quad T_{\mathbf{k}\mathbf{p}} T_{\mathbf{k}'\mathbf{p}'} \langle \tilde{T} \{ c_{\mathbf{k}\alpha}^\dagger(\tau) d_{\mathbf{p}\sigma}(\tau) c_{\mathbf{k}'\sigma_2}^\dagger(0) d_{\mathbf{p}'\sigma_1}(0) \} \rangle.\end{aligned}\quad (\text{A5})$$

Here, \tilde{T} denotes the time-ordering operator, and $\beta = 1/k_B T$ is the inverse temperature. Only $\mathbf{k}' = (-)\mathbf{k}$, $\mathbf{p}' = (-)\mathbf{p}$ contributes in the single-particle (two-particle) channel, while the diagonalized basis $\tilde{\varphi}_{\mathbf{k}\sigma}$ dictates that only $\sigma_2 = \alpha$, $\sigma_1 = \sigma$ contributes in the spin summation. Making use of the relation $\tilde{\phi}_{\mathbf{k}\sigma}^\dagger = \phi_{\mathbf{k}\sigma}^\dagger \hat{U}_{\mathbf{k}\sigma}$, Eq. (A5) becomes

$$\begin{aligned}\tilde{\Phi}_{\text{sp},\alpha\beta}(1\tilde{\omega}_\nu) &= \int_0^\beta d\tau e^{i\tilde{\omega}_\nu\tau} \sum_{\mathbf{k}\mathbf{p}\sigma} \hat{\mathcal{D}}_{\sigma\beta}^{(1/2)}(\vartheta) \hat{\mathcal{D}}_{\sigma\alpha}^{(1/2)}(\vartheta) T_{\mathbf{k}\mathbf{p}} T_{\mathbf{k}\mathbf{p}}^* \\ &\quad \times \langle \tilde{T} \{ [\hat{U}_{11\mathbf{k}\alpha}^* \gamma_{\mathbf{k}\alpha}^\dagger(\tau) + \hat{U}_{12\mathbf{k}\alpha}^* \gamma_{-\mathbf{k}\alpha}(\tau)] \\ &\quad \times [\hat{U}_{11\mathbf{k}\alpha} \gamma_{\mathbf{k}\alpha}(0) + \hat{U}_{12\mathbf{k}\alpha} \gamma_{-\mathbf{k}\alpha}^\dagger(0)] \} \rangle \\ &\quad \times \langle \tilde{T} [\hat{U}_{11\mathbf{p}\sigma} \gamma_{\mathbf{p}\sigma}(\tau) + \hat{U}_{12\mathbf{p}\sigma} \gamma_{-\mathbf{p}\sigma}^\dagger(\tau)] \\ &\quad \times [\hat{U}_{11\mathbf{p}\sigma}^* \gamma_{\mathbf{k}\sigma}^\dagger(0) + \hat{U}_{12\mathbf{p}\sigma}^* \gamma_{-\mathbf{k}\sigma}(0)] \} \rangle\end{aligned}\quad (\text{A6})$$

$$\begin{aligned}\tilde{\Phi}_{\text{tp},\alpha\beta}(1\tilde{\omega}_\nu) &= - \int_0^\beta d\tau e^{i\tilde{\omega}_\nu\tau} \sum_{\mathbf{k}\mathbf{p}\sigma} \hat{\mathcal{D}}_{\sigma\beta}^{(1/2)}(\vartheta) \hat{\mathcal{D}}_{\sigma\alpha}^{(1/2)}(\vartheta) T_{\mathbf{k}\mathbf{p}} T_{-\mathbf{k},-\mathbf{p}} \\ &\quad \times \langle \tilde{T} \{ [\hat{U}_{11\mathbf{k}\alpha}^* \gamma_{\mathbf{k}\sigma}^\dagger(\tau) + \hat{U}_{12\mathbf{k}\alpha}^* \gamma_{-\mathbf{k}\sigma}(\tau)] \\ &\quad \times [\hat{U}_{21\mathbf{k}\alpha} \gamma_{\mathbf{k}\sigma}(0) + \hat{U}_{22\mathbf{k}\alpha} \gamma_{-\mathbf{k}\sigma}^\dagger(0)] \} \rangle \\ &\quad \times \langle \tilde{T} [\hat{U}_{21\mathbf{p}\sigma}^* \gamma_{\mathbf{p}\sigma}^\dagger(0) + \hat{U}_{22\mathbf{p}\sigma}^* \gamma_{-\mathbf{p}\sigma}(0)] \\ &\quad \times [\hat{U}_{11\mathbf{p}\sigma} \gamma_{\mathbf{p}\sigma}(\tau) + \hat{U}_{12\mathbf{p}\sigma} \gamma_{-\mathbf{p}\sigma}^\dagger(\tau)] \} \rangle\end{aligned}\quad (\text{A7})$$

Since our diagonalized Hamiltonian has the form of a free-electron gas, *i.e.*

$$H_{\text{FMSC}} = \tilde{H}_0 + \sum_{\mathbf{k}\sigma} E_{\mathbf{k}\sigma} \gamma_{\mathbf{k}\sigma}^\dagger \gamma_{\mathbf{k}\sigma} \quad (\text{A8})$$

with $\tilde{H}_0 = H_0 - (E_{\mathbf{k}\uparrow} + E_{\mathbf{k}\downarrow})$, the product of the new fermion operators $\tilde{\varphi}_{\mathbf{k}\sigma}$ in Eq. (A6) yield unperturbed Green's functions according to

$$G_\alpha(\mathbf{k}, \tau - \tau') = \langle \tilde{T} \{ c_{\mathbf{p}\alpha}^\dagger(\tau') c_{\mathbf{k}\alpha}(\tau) \} \rangle \quad (\text{A9})$$

We then Fourier-transform Eq. (A9) into

$$G_\alpha(\mathbf{k}, \tau) = \frac{1}{\beta} \sum_{\omega_m} e^{-i\omega_m\tau} G_\alpha(\mathbf{p}, i\omega_m), \quad (\text{A10})$$

where $\omega_m = (2m+1)\pi/\beta$, $m = 1, 2, 3 \dots$ is a fermionic Matsubara frequency. The frequency summation over m is evaluated by contour integration as in *e.g.* Ref. 66 to yield the result

$$\begin{aligned}\frac{1}{\beta} \sum_m G_\alpha(\mathbf{k}, i\omega_m) G_\sigma(\mathbf{p}, i\tilde{\omega}_\nu + i\omega_m) &= \frac{f(E_{\mathbf{k}\alpha}) - f(E_{\mathbf{p}\sigma})}{i\tilde{\omega}_\nu + E_{\mathbf{k}\alpha} - E_{\mathbf{p}\sigma}} \\ \frac{1}{\beta} \sum_m G_\alpha(\mathbf{k}, i\omega_m) G_\sigma(\mathbf{p}, i\tilde{\omega}_\nu - i\omega_m) &= \frac{f(E_{\mathbf{p}\sigma}) - f(-E_{\mathbf{k}\alpha})}{i\tilde{\omega}_\nu - E_{\mathbf{k}\alpha} - E_{\mathbf{p}\sigma}},\end{aligned}\quad (\text{A11})$$

where $f(E) = 1 - f(-E) = 1/(1 + e^{\beta E})$ is the Fermi distribution. It is then a matter of straight-forward calculations to obtain the result

$$\begin{aligned}\Phi_{\text{sp},\alpha\beta}(-eV) &= \sum_{\mathbf{k}\mathbf{p}\sigma} \hat{\mathcal{D}}_{\sigma\beta}^{(1/2)}(\vartheta) \hat{\mathcal{D}}_{\sigma\alpha}^{(1/2)}(\vartheta) T_{\mathbf{k}\mathbf{p}} T_{\mathbf{k}\mathbf{p}}^* N_{\mathbf{k}\alpha}^2 N_{\mathbf{p}\sigma}^2 \\ &\quad \left[\frac{|\Delta_{\mathbf{k}\alpha\alpha} \Delta_{\mathbf{p}\sigma\sigma}|^2 \Lambda_{\mathbf{k}\mathbf{p}\sigma\alpha}^{-1,-1}(-eV)}{(\xi_{\mathbf{k}\alpha} + E_{\mathbf{k}\alpha})(\xi_{\mathbf{p}\sigma} + E_{\mathbf{p}\sigma})} + \Lambda_{\mathbf{k}\mathbf{p}\sigma\alpha}^{1,1}(-eV) \right. \\ &\quad \left. + \frac{|\Delta_{\mathbf{p}\sigma\sigma}|^2 \Lambda_{\mathbf{k}\mathbf{p}\sigma\alpha}^{-1,1}(-eV)}{\xi_{\mathbf{p}\sigma} + E_{\mathbf{p}\sigma}} \right. \\ &\quad \left. + \frac{|\Delta_{\mathbf{k}\alpha\alpha}|^2 \Lambda_{\mathbf{k}\mathbf{p}\sigma\alpha}^{1,-1}(-eV)}{\xi_{\mathbf{k}\alpha} + E_{\mathbf{k}\alpha}} \right]\end{aligned}\quad (\text{A12})$$

$$\begin{aligned}\Phi_{\text{tp},\alpha\beta}(eV) &= - \sum_{\mathbf{k}\mathbf{p}\sigma} \hat{\mathcal{D}}_{\sigma\beta}^{(1/2)}(\vartheta) \hat{\mathcal{D}}_{\sigma\alpha}^{(1/2)}(\vartheta) T_{\mathbf{k}\mathbf{p}} T_{-\mathbf{k},-\mathbf{p}} \\ &\quad \times \frac{\Delta_{\mathbf{k}\alpha\alpha}^* \Delta_{\mathbf{p}\sigma\sigma}}{4E_{\mathbf{k}\alpha} E_{\mathbf{p}\sigma}} \sum_{\substack{\lambda=\pm 1 \\ \rho=\pm 1}} \Lambda_{\mathbf{k}\mathbf{p}\sigma\alpha}^{\lambda\rho}(eV),\end{aligned}\quad (\text{A13})$$

where $\Lambda_{\mathbf{k}\mathbf{p}\sigma\alpha}^{\lambda\rho}(eV)$ is obtained by performing analytical continuation $i\tilde{\omega}_\nu \rightarrow eV + i0^+$ on

$$\tilde{\Lambda}_{\mathbf{k}\mathbf{p}\sigma\alpha}^{\lambda\rho}(i\tilde{\omega}_\nu) = \frac{\lambda[f(E_{\mathbf{k}\alpha}) - f(\lambda\rho E_{\mathbf{p}\sigma})]}{i\tilde{\omega}_\nu + \rho E_{\mathbf{k}\alpha} - \lambda E_{\mathbf{p}\sigma}}; \quad \lambda, \rho = \pm 1. \quad (\text{A14})$$

We also provide the details of the persistent spin-supercurrent for $\Delta_{\sigma\sigma} = 0$. Writing the Josephson current Eq. (32) out explicitly, one has that $I_J^C = eI^+$ and $I_J^S = -I^-$ where

$$\begin{aligned}
I^\pm = \sum_{\mathbf{k}\mathbf{p}} |T_{\mathbf{k}\mathbf{p}}|^2 & \left[\cos^2(\vartheta/2) \frac{|\Delta_{\mathbf{k}\uparrow\uparrow}\Delta_{\mathbf{p}\uparrow\uparrow}|}{E_{\mathbf{k}\uparrow}E_{\mathbf{p}\uparrow}} \sin \Delta\theta_{\uparrow\uparrow} F_{\mathbf{k}\mathbf{p}\uparrow\uparrow} \right. \\
& + \sin^2(\vartheta/2) \frac{|\Delta_{\mathbf{k}\uparrow\uparrow}\Delta_{\mathbf{p}\downarrow\downarrow}|}{E_{\mathbf{k}\uparrow}E_{\mathbf{p}\downarrow}} \sin(\theta_{\downarrow\downarrow}^L - \theta_{\uparrow\uparrow}^R) F_{\mathbf{k}\mathbf{p}\uparrow\downarrow} \\
& \pm \sin^2(\vartheta/2) \frac{|\Delta_{\mathbf{k}\downarrow\downarrow}\Delta_{\mathbf{p}\uparrow\uparrow}|}{E_{\mathbf{k}\downarrow}E_{\mathbf{p}\uparrow}} \sin(\theta_{\uparrow\uparrow}^L - \theta_{\downarrow\downarrow}^R) F_{\mathbf{k}\mathbf{p}\downarrow\uparrow} \\
& \left. \pm \cos^2(\vartheta/2) \frac{|\Delta_{\mathbf{k}\downarrow\downarrow}\Delta_{\mathbf{p}\downarrow\downarrow}|}{E_{\mathbf{k}\downarrow}E_{\mathbf{p}\downarrow}} \sin \Delta\theta_{\downarrow\downarrow} F_{\mathbf{k}\mathbf{p}\downarrow\downarrow} \right]. \quad (\text{A15})
\end{aligned}$$

The first and fourth term above vanish when $\Delta\theta_{\sigma\sigma} = 0$. By observing that $F_{\mathbf{k}\mathbf{p}\uparrow\downarrow} = F_{\mathbf{p}\mathbf{k}\downarrow\uparrow}$, we are then able to re-write Eq. (A15) as

$$\begin{aligned}
I^\pm = \sum_{\mathbf{k}\mathbf{p}} |T_{\mathbf{k}\mathbf{p}}|^2 \sin^2(\vartheta/2) & \frac{|\Delta_{\mathbf{k}\uparrow}\Delta_{\mathbf{p}\downarrow}|}{E_{\mathbf{k}\uparrow}E_{\mathbf{p}\downarrow}} F_{\mathbf{k}\mathbf{p}\uparrow\downarrow} \\
& \times \left[\sin(\theta_{\downarrow}^L - \theta_{\uparrow}^R) \pm \sin(\theta_{\uparrow}^L - \theta_{\downarrow}^R) \right] \\
= e \sum_{\mathbf{k}\mathbf{p}} |T_{\mathbf{k}\mathbf{p}}|^2 \sin^2(\vartheta/2) & \frac{|\Delta_{\mathbf{k}\uparrow}\Delta_{\mathbf{p}\downarrow}|}{E_{\mathbf{k}\uparrow}E_{\mathbf{p}\downarrow}} F_{\mathbf{k}\mathbf{p}\uparrow\downarrow} \\
& \times \left[\sin \left((\theta_{\downarrow}^L \mp \theta_{\uparrow}^R - \theta_{\uparrow}^R \pm \theta_{\downarrow}^L)/2 \right) \right. \\
& \left. \times \cos \left((\theta_{\downarrow}^L \pm \theta_{\uparrow}^R - \theta_{\uparrow}^R \mp \theta_{\downarrow}^L)/2 \right) \right]. \quad (\text{A16})
\end{aligned}$$

It is clear that the argument of the sine gives 0 for the upper

sign, such that $I_J^C = 0$. But for the lower sign, the argument of the cosine is equal to 0, such that Eq. (37) is obtained.

2. Ferromagnets with spin-orbit coupling

The spin-current across the junction can be written as

$$\begin{aligned}
\mathbf{I}^S &= \Im \{ \Phi(-eV) \}, \\
\Phi(-eV) &= \lim_{i\tilde{\omega}_\nu \rightarrow -eV + i0^+} \tilde{\Phi}(i\tilde{\omega}_\nu), \quad (\text{A17})
\end{aligned}$$

where we have defined the Matsubara function

$$\begin{aligned}
\tilde{\Phi}(i\tilde{\omega}_\nu) &= \sum_{\mathbf{k}\mathbf{p}\alpha\beta\sigma} |T_{\mathbf{k}\mathbf{p}}|^2 \hat{\sigma}_{\alpha\beta} \int_0^\beta d\tau e^{i\tilde{\omega}_\nu \tau} \\
&\quad \times \langle T \{ c_{\mathbf{k}\sigma}(0) c_{\mathbf{k}\alpha}^\dagger(\tau) \} \rangle \langle T \{ d_{\mathbf{p}\beta}(\tau) d_{\mathbf{p}\sigma}^\dagger(0) \} \rangle. \quad (\text{A18})
\end{aligned}$$

In Eq. (A18), we defined the time-ordering operator T while β in the upper integration limit is inverse temperature and $\tilde{\omega}_\nu = 2n\pi/\beta$, $n = 0, 1, 2, \dots$ is a bosonic Matsubara frequency. From the definition of the spin-generalized Green's function

$$G_{\mathbf{k}}^{\alpha\beta}(\tau - \tau') = -\langle T \{ c_{\mathbf{k}\alpha}(\tau) c_{\mathbf{k}\beta}^\dagger(\tau') \} \rangle, \quad (\text{A19})$$

Eq. (A18) can be written out explicitly to yield

$$\begin{aligned}
\tilde{\Phi}(i\tilde{\omega}_\nu) &= \frac{1}{\beta} \sum_{\mathbf{k}\mathbf{p},m} |T_{\mathbf{k}\mathbf{p}}|^2 \left[\hat{\sigma}_{\uparrow\uparrow} \left(G_{\mathbf{k}}^{\uparrow\uparrow}(i\omega_m) G_{\mathbf{p}}^{\uparrow\uparrow}(i\omega_m - i\tilde{\omega}_\nu) + G_{\mathbf{k}}^{\downarrow\uparrow}(i\omega_m) G_{\mathbf{p}}^{\uparrow\downarrow}(i\omega_m - i\tilde{\omega}_\nu) \right) \right. \\
&\quad + \hat{\sigma}_{\uparrow\downarrow} \left(G_{\mathbf{k}}^{\uparrow\downarrow}(i\omega_m) G_{\mathbf{p}}^{\downarrow\uparrow}(i\omega_m - i\tilde{\omega}_\nu) + G_{\mathbf{k}}^{\downarrow\downarrow}(i\omega_m) G_{\mathbf{p}}^{\downarrow\downarrow}(i\omega_m - i\tilde{\omega}_\nu) \right) \\
&\quad + \hat{\sigma}_{\downarrow\uparrow} \left(G_{\mathbf{k}}^{\downarrow\uparrow}(i\omega_m) G_{\mathbf{p}}^{\uparrow\downarrow}(i\omega_m - i\tilde{\omega}_\nu) + G_{\mathbf{k}}^{\uparrow\downarrow}(i\omega_m) G_{\mathbf{p}}^{\downarrow\uparrow}(i\omega_m - i\tilde{\omega}_\nu) \right) \\
&\quad \left. + \hat{\sigma}_{\downarrow\downarrow} \left(G_{\mathbf{k}}^{\downarrow\downarrow}(i\omega_m) G_{\mathbf{p}}^{\downarrow\downarrow}(i\omega_m - i\tilde{\omega}_\nu) + G_{\mathbf{k}}^{\uparrow\downarrow}(i\omega_m) G_{\mathbf{p}}^{\uparrow\downarrow}(i\omega_m - i\tilde{\omega}_\nu) \right) \right]. \quad (\text{A20})
\end{aligned}$$

We made use of the Fourier-transformations

$$\begin{aligned}
G_{\mathbf{k}}^{\alpha\beta}(i\omega_m) &= \int_0^\beta d\tau e^{i\omega_m \tau} G_{\mathbf{k}}^{\alpha\beta}(\tau), \\
G_{\mathbf{k}}^{\alpha\beta}(\tau) &= \frac{1}{\beta} \sum_m e^{-i\omega_m \tau} G_{\mathbf{k}}^{\alpha\beta}(i\omega_m) \quad (\text{A21})
\end{aligned}$$

in writing down Eq. (A20), where $\omega_m = 2(m+1)\pi/\beta$, $m = 0, 1, 2, \dots$ is a fermionic Matsubara frequency. Having written down the full expression for the Matsubara function in Eq. (A20), one can now easily distinguish between components of the spin-current. For instance, only $\hat{\sigma}_{\alpha\alpha}$ will contribute to the \hat{z} -component of \mathbf{I}^S , and the corresponding terms can be read out from Eq. (A20). From the present Green functions in Eq.

(50), it is obvious that three types of frequency summations must be performed, namely

$$\begin{aligned}
J_{\mathbf{k}\mathbf{p},r} &= \frac{1}{\beta} \sum_m \left[\frac{\omega_m^r}{[(\varepsilon_{\mathbf{k}\uparrow} - i\omega_m)(\varepsilon_{\mathbf{k}\downarrow} - i\omega_m) - y_{\mathbf{k}}^2]} \right. \\
&\quad \left. \times \frac{1}{[(\varepsilon_{\mathbf{p}\uparrow} - i\omega_m + i\tilde{\omega}_\nu)(\varepsilon_{\mathbf{p}\downarrow} - i\omega_m + i\tilde{\omega}_\nu) - y_{\mathbf{p}}^2]} \right], \quad (\text{A22})
\end{aligned}$$

with r is an integer. Performing the summation over m using residue calculus, one finds that

$$J_{\mathbf{k}\mathbf{p},r} = \sum_{\substack{\alpha=\pm \\ \beta=\pm}} \frac{\alpha\beta}{4y_{\mathbf{k}}y_{\mathbf{p}}} \left[\frac{\psi_{\mathbf{k}\alpha}^r n(\psi_{\mathbf{k}\alpha}) - (i\tilde{\omega}_{\nu} + \psi_{\mathbf{p}\beta})^r n(\psi_{\mathbf{p}\beta})}{-i\tilde{\omega}_{\nu} + \psi_{\mathbf{k}\alpha} - \psi_{\mathbf{p}\beta}} \right] \quad (\text{A23})$$

with the definition $\psi_{\mathbf{k}\alpha} \equiv \varepsilon_{\mathbf{k}} + \alpha y_{\mathbf{k}}$. Separating the general expression Eq. (A20) into its spatial components $\tilde{\Phi} = (\tilde{\Phi}_x, \tilde{\Phi}_y, \tilde{\Phi}_z)$, the components of the spin-current can be extracted according to $I_i^S = \Im\{\Phi_i(-eV)\}$, $i = x, y, z$. Note that the charge-current in this model, which vanishes for $eV = 0$, is obtained by the performing the replacement $\hat{\sigma}_{\alpha\beta} \rightarrow \hat{1}_{\alpha\beta}$, where $\hat{1}$ is the 2×2 unit matrix. We find that

$$\begin{aligned} \tilde{\Phi}_x(i\tilde{\omega}_{\nu}) &= \sum_{\mathbf{k}\mathbf{p}} \frac{|T_{\mathbf{k}\mathbf{p}}|^2}{4\gamma_{\mathbf{k}}\gamma_{\mathbf{p}}} \left[J_{\mathbf{k}\mathbf{p},0} \left(\varepsilon_{\mathbf{k}\downarrow}(\zeta_{\mathbf{L}} - B_{\mathbf{p},-}) + (\varepsilon_{\mathbf{p}\uparrow} + i\tilde{\omega}_{\nu})(\zeta_{\mathbf{R}} - B_{\mathbf{k},-}) + (\varepsilon_{\mathbf{p}\downarrow} + i\tilde{\omega}_{\nu})(\zeta_{\mathbf{R}}^{\dagger} - B_{\mathbf{k},+}) + \varepsilon_{\mathbf{k}\uparrow}(\zeta_{\mathbf{L}}^{\dagger} - B_{\mathbf{p},-}) \right) \right. \\ &\quad \left. - J_{\mathbf{k}\mathbf{p},1} \left((\zeta_{\mathbf{L}} - B_{\mathbf{p},-}) + (\zeta_{\mathbf{R}} - B_{\mathbf{k},-}) + (\zeta_{\mathbf{R}}^{\dagger} - B_{\mathbf{k},+}) + (\zeta_{\mathbf{L}}^{\dagger} - B_{\mathbf{p},+}) \right) \right], \\ \tilde{\Phi}_y(i\tilde{\omega}_{\nu}) &= \sum_{\mathbf{k}\mathbf{p}} i \frac{|T_{\mathbf{k}\mathbf{p}}|^2}{4\gamma_{\mathbf{k}}\gamma_{\mathbf{p}}} \left[J_{\mathbf{k}\mathbf{p},0} \left(-\varepsilon_{\mathbf{k}\downarrow}(\zeta_{\mathbf{L}} - B_{\mathbf{p},-}) - (\varepsilon_{\mathbf{p}\uparrow} + i\tilde{\omega}_{\nu})(\zeta_{\mathbf{R}} - B_{\mathbf{k},-}) + (\varepsilon_{\mathbf{p}\downarrow} + i\tilde{\omega}_{\nu})(\zeta_{\mathbf{R}}^{\dagger} - B_{\mathbf{k},+}) + \varepsilon_{\mathbf{k}\uparrow}(\zeta_{\mathbf{L}}^{\dagger} - B_{\mathbf{p},-}) \right) \right. \\ &\quad \left. - J_{\mathbf{k}\mathbf{p},1} \left(-(\zeta_{\mathbf{L}} - B_{\mathbf{p},-}) - (\zeta_{\mathbf{R}} - B_{\mathbf{k},-}) + (\zeta_{\mathbf{R}}^{\dagger} - B_{\mathbf{k},+}) + (\zeta_{\mathbf{L}}^{\dagger} - B_{\mathbf{p},+}) \right) \right], \\ \tilde{\Phi}_z(i\tilde{\omega}_{\nu}) &= \sum_{\mathbf{k}\mathbf{p}} \frac{|T_{\mathbf{k}\mathbf{p}}|^2}{4\gamma_{\mathbf{k}}\gamma_{\mathbf{p}}} \left[J_{\mathbf{k}\mathbf{p},0} \left(\varepsilon_{\mathbf{k}\downarrow}(\varepsilon_{\mathbf{p}\downarrow} + i\tilde{\omega}_{\nu}) - \varepsilon_{\mathbf{k}\uparrow}(\varepsilon_{\mathbf{p}\uparrow} + i\tilde{\omega}_{\nu}) + (\zeta_{\mathbf{R}} - B_{\mathbf{k},-})(\zeta_{\mathbf{L}}^{\dagger} - B_{\mathbf{p},+}) - (\zeta_{\mathbf{R}}^{\dagger} - B_{\mathbf{k},+})(\zeta_{\mathbf{L}} - B_{\mathbf{p},-}) \right) \right. \\ &\quad \left. + J_{\mathbf{k}\mathbf{p},1} \left(\varepsilon_{\mathbf{k}\uparrow} - \varepsilon_{\mathbf{k}\downarrow} + \varepsilon_{\mathbf{p}\uparrow} - \varepsilon_{\mathbf{p}\downarrow} \right) \right]. \quad (\text{A24}) \end{aligned}$$

- ¹ I. Zutic, J. Fabian, and S. D. Sarma, Rev. Mod. Phys. **76**, 323 (2004).
- ² M. I. D'yakonov, V. I. Perel, Phys. Lett. A **35**, 459 (1971).
- ³ J. Wunderlich, B. Kaestner, J. Sinova, and T. Jungwirth, Phys. Rev. Lett. **94**, 047204 (2005).
- ⁴ J. Tallon, C. Bernhard, M. Bowden, P. Gilberd, T. Stoto, and D. Pringle, IEEE, Trans. Appl. Supercond. **9**, 1696 (1999).
- ⁵ S. S. Saxena, P. Agarwal, K. Ahilan, F. M. Grosche, R. K. W. Haselwimmer, M. J. Steiner, E. Pugh, I. R. Walker, S. R. Julian, and P. Monthoux, Nature **406**, 587 (2000).
- ⁶ D. Aoki, A. Huxley, E. Ressouche, D. Braihwaite, J. Flouquet, J.-P. Brison, E. Lhotel, and C. Paulsen, Nature **413**, 613 (2001).
- ⁷ V. L. Ginzburg, Sov. Phys. JETP **4**, 153 (1957).
- ⁸ R. Shen, Z. M. Zheng, S. Liu, and D. Y. Xing, Phys. Rev. B **67**, 024514 (2003).
- ⁹ P. Fulde and R. A. Ferrel, Phys. Rev. **135**, A550 (1964); A. I. Larkin and Y. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **47**, 1136 (1964) [Sov. Phys. JETP **20**, 762 (1965)].
- ¹⁰ M. B. Walker and K. V. Samokhin, Phys. Rev. Lett. **88**, 207001 (2002).
- ¹¹ K. Machida and T. Ohmi, Phys. Rev. Lett. **86**, 850 (2001).
- ¹² K. Ishida, H. Mukuda, Y. Kitaoka, K. Asayama, Z. Q. Mao, Y. Mori, and Y. Maeno, Nature **396**, 658 (1998).
- ¹³ K. D. Nelson, Z. Q. Mao, Y. Maeno, and Y. Liu, Science **306**, 1151 (2004).

- ¹⁴ A. Brataas and Y. Tserkovnyak, Phys. Rev. Lett. **93**, 087201 (2004).
- ¹⁵ Y. Tanaka and S. Kashiwaya, Phys. Rev. B **70**, 012507 (2004).
- ¹⁶ T. Koyama and M. Tachiki, Phys. Rev. B **30**, 6463 (1984).
- ¹⁷ M. Grønsløth, J. Linder, J.-M. Børven, and A. Sudbø, Phys. Rev. Lett. **97**, 147002 (2006).
- ¹⁸ D. V. Shopova and D. I. Uzunov, Phys. Rev. B **72**, 024531 (2005).
- ¹⁹ M. L. Kulić, C. R. Physique **7**, 4 (2006); M. L. Kulić, and I. M. Kulić, Phys. Rev. B **63**, 104503 (2001).
- ²⁰ I. Eremin, F. S. Nogueira, and R.-J. Tarento, Phys. Rev. B **73**, 054507 (2006).
- ²¹ T. Dietl, Semicond. Sci. Technol. **17**, 377 (2002).
- ²² F. Matsukura, H. Ohno, and T. Dietl, Handbook of Magnetic Materials, vol. 14 (Elsevier, 2002).
- ²³ H.-A. Engel, E. I. Rashba, and B. I. Halperin, cond-mat/0603306.
- ²⁴ F. S. Nogueira and K.-H. Bennemann, Europhys. Lett. **67**, 620 (2004).
- ²⁵ Y.-L. Lee and Y.-W. Lee, Phys. Rev. B **68**, 184413 (2003).
- ²⁶ J. C. Slonczewski, Phys. Rev. B **39**, 6995 (1989).
- ²⁷ T. Yokoyama, Y. Tanaka, and J. Inoue, Phys. Rev. B **72**, 220504 (2005).
- ²⁸ J. Wang and K. S. Chan, cond-mat/0512430.
- ²⁹ S. I. Kiselev, J. C. Sankey, I. N. Krivorotov, N. C. Emley, R. J. Schoelkopf, R. A. Buhrman, and D. C. Ralph, Nature **425**, 380 (2003).

- ³⁰ G. E. W. Bauer, A. Brataas, Y. Tserkovnyak, and B. J. van Wees, *Appl. Phys. Lett.* **82**, 3928 (2003).
- ³¹ A. Brataas, Y. Tserkovnyak, G. E. W. Bauer, and B. I. Halperin, *Phys. Rev. B* **66**, 060404 (2002).
- ³² J. E. Hirsch, *Phys. Rev. Lett.* **83**, 1834 (1999).
- ³³ Y. Tserkovnyak, A. Brataas, and G. E. W. Bauer, *Phys. Rev. Lett.* **88**, 117601 (2002).
- ³⁴ S. Tewari, D. Belitz, T. R. Kirkpatrick, and J. Toner, *Phys. Rev. Lett.* **93**, 177002 (2004).
- ³⁵ V. P. Mineev, *cond-mat/0507572*.
- ³⁶ V. P. Mineev and K. V. Samokhin, *Introduction to Unconventional Superconductivity* (Gordon and Breach, New York, 1999).
- ³⁷ H. Kotegawa, A. Harada, S. Kawasaki, Y. Kawasaki, Y. Kitaoka, Y. Haga, E. Yamamoto, Y. Onuki, K. M. Itoh, and E. E. Haller, *J. Phys. Soc. Jpn.* **74**, 705 (2005).
- ³⁸ C. Bernhard, J. L. Tallon, E. Brucher, and R. K. Kremer, *Phys. Rev. B* **61**, R14960 (2000).
- ³⁹ C.-R. Hu, *Phys. Rev. Lett.* **72**, 1526 (1994).
- ⁴⁰ V. Ambegaokar, P. G. deGennes, and D. Rainer, *Phys. Rev. A* **9**, 2676 (1974).
- ⁴¹ L. J. Buchholtz and G. Zwicknagl, *Phys. Rev. B* **23**, 5788 (1981).
- ⁴² Y. Tanuma, Y. Tanaka, and S. Kashiwaya, *Phys. Rev. B* **64**, 214519 (2001).
- ⁴³ S. Datta and B. Das, *Appl. Phys. Lett.* **56**, 665 (1990).
- ⁴⁴ A. J. Leggett, *Rev. Mod. Phys.* **47**, 331 (1975).
- ⁴⁵ F. Hardy and A. D. Huxley, *Phys. Rev. Lett.* **94**, 247006 (2005).
- ⁴⁶ K. V. Samokhin and M. B. Walker, *Phys. Rev. B* **66**, 174501 (2002).
- ⁴⁷ M. H. Cohen, L. M. Falicov, and J. C. Phillips, *Phys. Rev. Lett.* **8**, 316 (1962).
- ⁴⁸ H. P. Dahal, J. Jackiewicz, and K. S. Bedell, *Phys. Rev. B* **72**, 172506 (2005).
- ⁴⁹ E. P. Wigner, *Gruppentheorie und ihre Anwendung auf die Quantenmechanik der Atomspektren* (Frieder. Vieweg, Braunschweig, 1931).
- ⁵⁰ J. Shi and Q. Niu, *cond-mat/0601531*.
- ⁵¹ C. Bruder, A. van Otterlo, and G. T. Zimanyi, *Phys. Rev. B* **51**, 12904 (1995).
- ⁵² H. Ohno, H. Munekata, T. Penney, S. von Molnár, and L. L. Chang, *Phys. Rev. Lett.* **68**, 2664 (1992).
- ⁵³ F. Matsukura, H. Ohno, A. Shen, and Y. Sugawara, *Phys. Rev. B* **57**, R2037 (1998).
- ⁵⁴ J. Kikkawa and D. Awschalom, *Nature* **397**, 139 (1999).
- ⁵⁵ T. Jungwirth, Q. Niu, and A. H. MacDonald, *Phys. Rev. Lett.* **88**, 207208 (2002).
- ⁵⁶ K. Børkje and A. Sudbø, *Phys. Rev. B* **74**, 054506 (2006).
- ⁵⁷ G. Dresselhaus, *Phys. Rev.* **100**, 580 (1955).
- ⁵⁸ A. G. Malshukov, C. S. Tang, C. S. Chu, and K. A. Chao, *Phys. Rev. Lett.* **95**, 107203 (2005).
- ⁵⁹ R. S. Keizer, S. T. B. Goennenwein, T. M. Klapwijk, G. Miao, G. Xiao, and A. Gupta, *Nature* **439**, 825 (2006).
- ⁶⁰ M. E. Simon and C. M. Varma, *Phys. Rev. Lett.* **89**, 247003 (2002).
- ⁶¹ J. Bass and W. P. Pratt Jr., *Journal of Magnetism and Magnetic Materials* **200**, 274 (1999).
- ⁶² P. Mohanty, G. Zolfagharkhani, S. Kettemann, and P. Fulde, *Phys. Rev. B* **70**, 195301 (2004).
- ⁶³ Q. Feng Sun, H. Guo, and J. Wang, *Phys. Rev. B* **69**, (2004).
- ⁶⁴ F. Meier and D. Loss, *Phys. Rev. Lett.* **90**, 167204 (2003).
- ⁶⁵ F. Schutz, M. Kollar, and P. Kopietz, *Phys. Rev. Lett.* **91**, 017205 (2003).
- ⁶⁶ G. D. Mahan, *Many-Particle Physics* (Kluwer Academic/Plenum Publishers, 2002), 3rd ed.
- ⁶⁷ Note that $N_{\alpha\beta}$ reduces to the number operator when we sum over equal spins, *i.e.* $N = \sum_{\sigma} N_{\sigma\sigma}$.
- ⁶⁸ For corresponding results in spin-singlet superconductors with helimagnetic order, see Refs. 19,20.
- ⁶⁹ Note that the index α on the quasi-particles does not denote the physical spin of electrons, but is rather to be considered as some unspecified helicity-index. The usage of the word “spin” in this context then refers to this helicity.